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COMPRESSION CHARACTERISTICS AND STRUCTURAL BEAM DESIGN ANALYSIS OF STEEL FIBER REINFORCED CONCRETE

ARMY CONSTRUCTION ENGINEERING RESEARCH LABORATORY

DECEMBER 1973

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This report studies the effect of steel fibers on the static compression strength of concrete and mortar. Using 6 × 12 in., 4 × 8 in., and 3 × 6 in. cylinders, 180 tests were conducted on three different mixes: sand, ⅜ in. maximum aggregate, and ¾ in. maximum aggregate. Using 0.0010 × 0.022 × 1.0 in. fibers in volume percentages of 1.0, 1.5, 2.0, and 2.5 in a procedure to eliminate any strength variation in the matrix, it was found that there was a slight decrease in the compressive strength for the sand

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mix. A 16 and 23 percent increase in strength was found for the 3/8 in. and 3/4 in. maximum aggregate mixes respectively. Data that indicate the strength of the fiber concrete may vary inversely with the sand content. Values for Young's modulus and Poisson's ratio for the various fiber percentages are also given. It is suggested that fibers can be used economically in flexural members. A cost comparison of a reinforced concrete beam with stirrups versus a reinforced concrete beam with fibers with no stirrups is presented.

FOREWORD

This investigation was sponsored by the U.S. Army Corps of Engineers, Office of the Chief of Engineers (OCE), Washington, D.C. as part of two Research, Development, Test, and Evaluation (RDT&E) Programs. One part of the study, for the Directorate of Military Construction, was conducted under Project 4A664717D895, "Military Construction Systems Development," Task 04, "Military Airfield Facilities," Work Unit 008, "Applicability of Fibrous Concrete for Military Facilities" (OCE Technical Coordinator L. Price). The other, for the Directorate of Military Engineers, was conducted under Project 4A664717D895, Task 23, "Construction Effort Analyses," Work Unit 003, "Application of Fibrous Concrete for Construction in the Theater of Operation."

This investigation was conducted by the Materials Division of the U.S. Army Construction Engineering Research Laboratory (CERL), Champaign, Illinois, from July 1972 to January 1973. CERL personnel actively engaged in the investigation were Dr. G. R. Williamson, on a summer academic appointment from Youngstown State University, Youngstown, Ohio, and B. H. Gray, principal investigator for the work units.

During this investigation, COL R. W. Reisacher was Director of CERL and Dr. L. R. Shaffer was Deputy Director. E. A. Lotz was Chief of the Materials Division.

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COMPRESSION CHARACTERISTICS AND STRUCTURAL BEAM DESIGN ANALYSIS OF STEEL FIBER REINFORCED CONCRETE

1 INTRODUCTION

Problem. Fiber-reinforced concrete is ordinary concrete with randomly dispersed fibers (usually steel) of short length and small diameter.[1] During the past decade, steel fiber-reinforced concrete has been widely researched by the Corps of Engineers and others. Most of the research has been directed toward improving the tensile and flexural strength of concrete because of the inherent weakness of plain concrete in these respects. Previous studies have dealt with fatigue, impact resistance, freeze-thaw durability, and compressive strength. The enhancement of these characteristics by the inclusion of steel fibers has been firmly established except in the area of compressive strength. Reports have varied from no increase in compressive strength due to the fibers, to as much as 100 percent increase in strength over the plain mix.[2,3,4] This inconsistency also applies to the effect of the fibers when used with mortar and when used with concrete. The inability of researchers to achieve similar results can be traced in part to lack of uniformity in preparation of the specimens. Because of conflicting data, the development of applications of fibrous concrete utilizing the compression characteristics has not paralleled the development of applications using the flexural characteristics.

Objective and Scope. The objective of this study was to determine the effect of steel fibers upon the static compression strength of concrete and mortar. These data are used with shear data reported by Romualdi and Ramey,[5] and Batson et al.,[6] to justify, from a compression standpoint, the use of steel fiber-reinforced concrete in conventionally reinforced flexural members such as beams. The effect of the fibers upon the ductility of the concrete, Young's modulus, and Poisson's ratio is discussed. Also discussed is the effect of the test cylinder size upon the ultimate compressive strength. A cost comparison is made between a conventionally reinforced beam with shear reinforcement and a similar beam with fibers replacing the shear reinforcement.

Background. Romualdi and Batson[7] theorize that the increase in flexural strength and ductility of concrete resulting from the use of steel fibers is attributable to the ability of the fibers to act as crack arrestors. Hsu et al.,[8] have shown that all concrete contains small flaws which begin to increase in size under stresses well below 50 percent of ultimate. The coalescing of these growing flaws ultimately results in the failure of the concrete. The randomly oriented steel fibers in fibrous concrete inhibit the growth of the flaws by the bond developed between the fiber and the matrix. Coarse aggregate has also been shown as a crack arrestor on the one hand and a crack initiator on the other. A growing crack, approaching a particle of coarse aggregate whose strength is greater than that of the matrix through which the crack is growing, must then detour around the particle. The pathway taken by the crack is usually along the interface between the aggregate and the matrix. The additional energy required for the longer path to overcome the bond at the interface requires an increase in load to permit continued growth of the crack. If, however, the bond between

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1 G. R. Williamson and B. H. Gray, *Technical Information Pamphlet on Use of Fibrous Concrete* (Construction Engineering Research Laboratory [CERL], May 1972).

2 G. R. Williamson, *Fibrous Reinforcement for Portland Cement Concrete*, Technical Report No. 2-40 (Ohio River Division Laboratories [ORDL], May 1965).

3 D. L. Birkimer and J. R. Hossley, *Comparison of Static and Dynamic Behavior of Plain and Fibrous Reinforced Concrete Cylinders*, Technical Report No. 4-69 (ORDL, January 1968).

4 Wai-Fah Chen and J. L. Carson, "Stress-Strain Properties of Random Wire Reinforced Concrete," *ACI Journal, Proceedings*, Vol 68, No. 12 (December 1971).

5 J. P. Romualdi and M. R. Ramey, *Effects of Impulsive Loads on Fiber-Reinforced Concrete Beams*, Final Report for the Office of Civil Defense, Contract No. OCO-PS-64-21 (Carnegie Institute of Technology, October 1965).

6 G. B. Batson, E. Jenkins, and R. Spatney, "Steel Fibers as Shear Reinforcement in Beams," *ACI Journal, Proceedings*, Vol 69 (October 1972).

7 J. P. Romualdi and G. B. Batson, "Mechanics of Crack Arrest in Concrete with Closely Spaced Reinforcement," *Proceedings American Society of Civil Engineers (ASCE)*, Vol 89 (June 1963).

8 T. T. C. Hsu, F. O. Slate, G. M. Sturman, and G. Winter, "Microcracking of Plain Concrete and the Shape of the Stress Strain Curve," *American Concrete Institute (ACI) Journal, Proceedings*, Vol 60 (February 1963).

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the matrix and aggregate particle has never been fully developed, then a flaw exists, and the aggregate particle acts as a crack initiator. Water trapped on the underside of an aggregate particle as it rises during the initial setting process will produce such a flaw. This function of acting as both a crack arrestor and crack initiator applies to the fibers as well.

The crack arrest theory, though not completely substantiated, does help to account for the increased flexural strength of steel fiber-reinforced concrete over plain concrete. To apply this concept to concrete in compression requires an understanding of the failure mechanism under compression loading. The failure of concrete cylinders in uniaxial compression is usually described as one of three types: a diagonal shear, a double shear resulting in a cone, and vertical splitting. A combination of the three is not unusual. This variation in failure mode has made the development of a compression strength theory for concrete rather difficult. Glucklich,[9] using the work of R. Jones and H. Neuber, has shown that these failure modes are not the primary ones, but are a result of the influence of the friction forces generated at the top and bottom of the test cylinders by the testing machine platens. The primary failure mode is a result of flaws or cracks which grow under load, parallel to the direction of the applied stress. These flaws or cracks convert the compressive stresses to tensile stresses in the vicinity of the crack tips. These tensile stresses cause the crack to grow until a critical size is reached, and a brittle type failure results. The failure mode is primarily tensile, therefore, the crack arrest theory applies. Some benefit should be gained from the use of steel fibers for concrete in compression.

2 METHOD

General. Three separate test series were conducted; for each series, 60 test cylinders were tested in uniaxial compression under static conditions. Fabrication and testing of all specimens was done in accordance with American Society for Testing and Materials (ASTM) Specifications. These specifications are listed in Appendix A.

To obviate size effects, three cylinder sizes were used: 3 × 6 in., 4 × 8 in., and 6 × 12 in., with 20 cylinders of each size for a total of 60 for each series. The fiber percentages used were 1.0, 1.5, 2.0, and 2.5 percent by volume of the cement, fine and coarse aggregate, and water. Of the 20 cylinders per size, this arrangement permitted four test specimens for each fiber percentage and for the plain control mix.

The mixing was done with a 3½ cu ft rotary drum mixer. Thirty specimens were made from each batch; two specimens for each size and each fiber percentage. For each of the series, the required two batches were made on successive days. The cylinders were allowed to set overnight, then removed from the molds and cured in saturated limewater for 11 days. They were then removed from the tank, air-dried, and tested at 28 days.

To prevent any variation in the strength of the matrix, the following mixing procedure was used. Enough material for 30 cylinders (cement, water, and aggregate) was thoroughly mixed. The concrete was then removed from the mixer and divided into five parts. The first part was used to make the plain cylinders, two each of the three sizes. The second part was then put back into the mixer and 2½ percent fiber added. The fibers were hand-sprinkled into the rotating drum. Mixing was continued for one minute after the fibers had been added. This procedure was then repeated for the 2.0, 1.5, and 1.0 percent fiber mixes, in that order, using the three remaining parts of the mix.

Materials. Type III cement from one source was used for the three test series. The fine aggregate was local sand, with 6.4 percent retained on a No. 4 sieve, and a fineness modulus of 3.0. The coarse aggregate was local crushed limestone. The ⅜ in. maximum size aggregate had a fineness modulus of 5.30, and the ¾ in. maximum size aggregate had a fineness modulus of 6.87. The sieve analysis for each of the aggregates is given in Table 1. Tap water was used for mixing. No additives of any type were used in any of the test series, nor was the water-cement ratio increased to improve workability.

The fiber used for the entire project was a 0.010 × 0.022 × 1.0 in. steel fiber manufactured from low-carbon steel plate by a chopping process. The fiber has a yield stress of 90,000 psi and an ultimate stress of 90-100,000 psi.

[9] J. Glucklich, "On the Compression Failure of Concrete," *Theoretical and Applied Mechanics*, Report No. 215 (University of Illinois, March 1962).

Table 1

Sieve Analysis of Fine and Coarse Aggregate

Sieve	Sand	Percent Retained 3/8"	3/4"
1 1/2	0	0	0
3/4	0	0	3.1
3/8	0	1.3	92.6
No. 4	6.7	55.2	98.5
8	22.9	91.3	98.5
16	36.6	93.5	98.5
30	52.5	94.8	98.5
50	82.5	96.4	98.5
100	99.6	97.5	98.5
F.M.	3.0	5.30	6.87

Mix Designs. The basic mixes for the three test series were designed to attain approximately 5,000 psi compressive strength at age 7 days. Work by Hsu and Slate[10] has shown that the tensile bond strength between paste and limestone aggregate (a reactive material) is time dependent, roughly paralleling the compressive strength development. (A reactive aggregate is one that contains cementious material which can react with the cement and water of the mix to further enhance bond development. Limestone is such a material, even as one of the ingredients of cement.) With crushed natural limestone and embedded Type I cement paste, the 28-day bond strength was found to increase 33 percent over the 7-day strength, while the compressive strength of the paste increased 50 percent during the same period. For an unreactive aggregate such as sandstone, the 28-day bond strength showed an increase of 21 percent over the 7-day strength. Steel fibers would of course be a nonreactive component of the fibrous concrete. Nevertheless, because the crack arrest mechanism is a function of the fiber-matrix bond, the curing time for this program was extended to 18 days and the test age to 28 days. This procedure plus the use of Type III cement assured maximum bond and compressive strength development.

The mix proportions for each test series are given in Table 2. Because of mixing difficulties with the first batch of the 3/4 in. maximum aggregate mix, the water-cement ratio was increased from 0.54 to 0.57 for the second batch. This resulted in a decrease in strength of approximately 20 percent.

Testing and Instrumentation. The cylinders were tested using a 500 kip Satec Universal Testing Machine and a 1000 kip Materials Testing System operating in load control. Two of the four 6 × 12 in. cylinders tested for each volume percent of fiber were used to obtain data for computing Young's modulus and Poisson's ratio. Strain data for determining the modulus was obtained from three 2-in. electrical resistance strain gages mounted vertically at the mid-height of the cylinder. The gages were equally spaced on the circumference at 120 degrees. The horizontal strain data was obtained from three 2 in. gages placed horizontally beneath the vertical gages. The output from each set of gages fed into an analog computer for averaging. The average value was recorded using an x-y plotter.

3 RESULTS AND DISCUSSION

General. The individual test results are tabulated in Appendix B. Listed in the tables are the ultimate

10 T. T. C. Hsu and F. O. Slate, "Tensile Bond Strength Between Aggregate and Cement Paste or Mortar," *ACI Journal, Proceedings*, Vol 60 (April 1963).

Table 2

Mix Proportions

Series Number	Cement Lbs.	Aggregate, Lbs. Sand	3/8" max.	3/4" max.	Water Lbs. W/C	Slump inches	Combined F.M.	Cylinder Compaction
I	794	2282	0	0	484, 0.61	6-7	3.00	Hand Rod
II	950	1368	912	0	522, 0.55	9	3.93	Vibrating Table
III	955	1375	0	917	516, 0.54	4	4.87	Vibrating Table
IIIa	938	1351	0	900	537, 0.57	6	4.87	Vibrating Table

strength of each cylinder with the standard deviation and coefficient of variation for each set of specimens. Also included are the calculated values of Young's modulus and Poisson's ratio.

Compressive Strength. The effect of the steel fibers upon the compressive strength of 6 × 12 in. cylinders of mortar and concrete is shown in Figure 1, a plot of strength versus fiber percentage. For mortar (Test Series No. 1, Table B1) fibers tend to decrease the ultimate strength slightly. The strength variation between the plain mortar and the mortar with 2.5 percent fiber is 6.5 percent. Although each of the fiber mortars shows some decrease in ultimate strength, it is not uniform. The amount of decrease shown for the mortar mix is insignificant. However, the fact that the fibers did not increase the strength is significant.

For the 3/8 in. maximum aggregate mix (Test Series No. 2, Table B2), the data of Figure 1 shows a considerable increase in compressive strength with increasing amount of fibers. For the 2.5 percent fiber mix, the strength is 16 percent greater than the plain mix.

The greatest increase in compressive strength resulting from the use of fibers occurred with the 3/4 in. maximum size aggregate mix (Test Series No. 3, Table B3). For this series, the 2.5 percent fiber mix was 23 percent stronger in compression than was the plain mix. Figures 2 and 3 are plots of strength versus fiber content for the 4 × 8 in. and the 3 × 6 in. cylinders respectively. The data shown in these figures are in agreement with that of Figure 1.

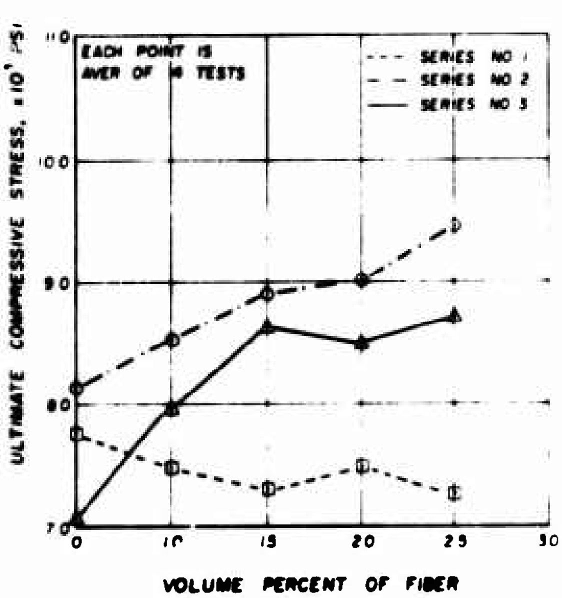

Figure 1. The effect of steel fibers on the compressive strength of 6 x 12 in. cylinders.

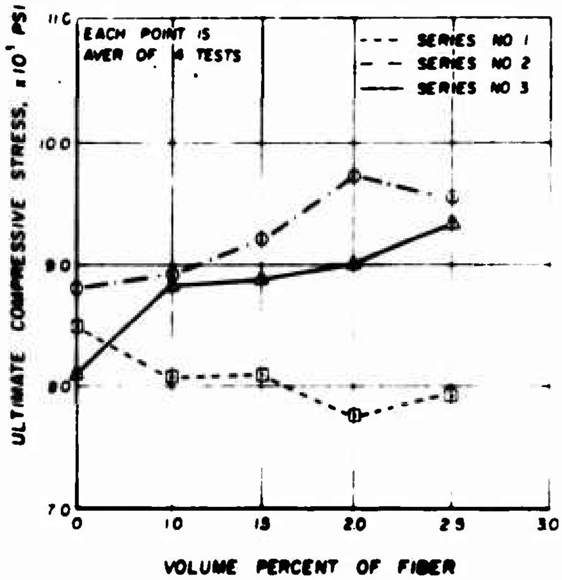

Figure 2. The effect of steel fibers on the compressive strength of 4 x 8 in. cylinders.

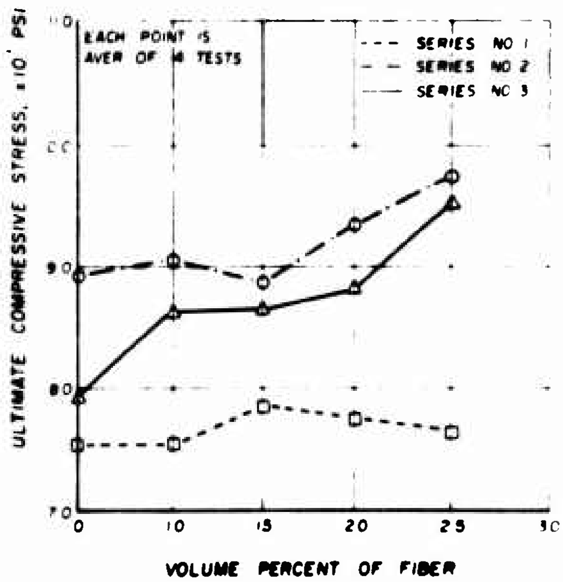

Figure 3. The effect of steel fibers on the compressive strength of 3 x 6 in. cylinders.

As is indicated in Table 1, the method used for compacting the cylinders for Test Series No. 1 (Table B1) was by hand rodding, while for Series Nos. 2 and 3 (Tables B2 and B3) they were compacted with the use of a Syntron Vibrating Table. Since this was the only variation in test procedures, a program was conducted to determine if the Series No. 1 results could have been affected by the compaction procedures. Using 4 × 8 in. cylinders and the mortar mix design as given in Table 1, four cylinders were then water-cured for 18 days and air-dried for 10 days before testing. The results listed in Table B4 show that the method of compacting the cylinders did not affect the results of Series No. 1.

Using the ratio of the compressive strength of the fiber mixes to that of the plain mix as one of the variables, Figure 4 shows the variation in compressive strength of 6 × 12 in. cylinders with respect to the theoretical fiber spacing. The equation used to compute the average spacing s is

$$s = 13.8d\sqrt{\frac{1}{p}}$$

where d is the fiber diameter and p is the volume percent of fiber. For the rectangular fiber used in this study, the fiber diameter d is simply taken as the diameter of an equivalent, circular, cross sectional area. For the 0.010 × 0.022 in. cross section, d is approximately 0.017 in.

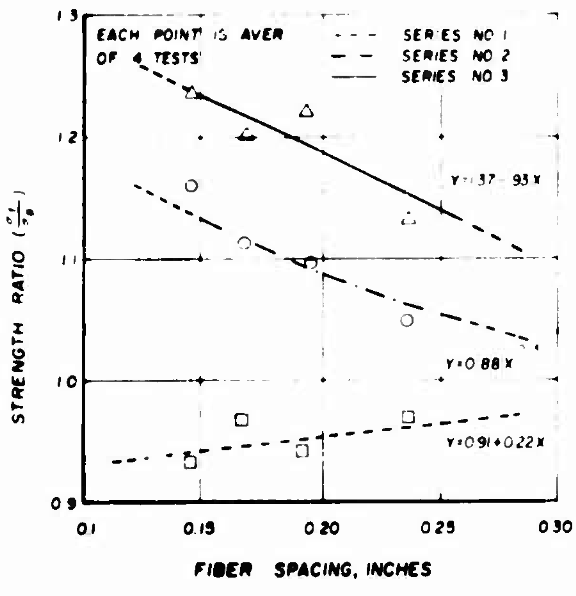

Figure 4. Strength ratio as a function of fiber spacing (6 x 12 in. cylinders).

If a 10 percent increase in compressive strength is arbitrarily accepted as significant, it is seen from Figure 4 that the fibers are effective at a spacing of 0.18 in. for the ⅜ in. mix and 0.24 in. for the ¾ in. mix. A spacing of 0.20 in. corresponds to a volume percentage of approximately of 1.5. For fiber concrete in bending, Williamson[11] indicates that an increase in flexural stress of 25 percent can be expected for this spacing. However, as was pointed out earlier, the fibers only reinforce the matrix, and if the bond between the mortar and the aggregate is fully developed, then the aggregate can also act as a crack arrestor. Hsu and Slate[12] reported a mortar-aggregate tensile bond strength of 329 psi and a paste-aggregate tensile bond strength of 414 psi for a good grade limestone. Williamson,[13] using single-fiber static pull out tests with -in. embedment, reported a mortar-fiber bond strength of 410 psi for a No. 8 maximum size aggregate mix. Based upon these values, it would seem that the coarse aggregate can function as a crack arrestor. To obtain a more realistic understanding of the effect of the fiber, the spacing should be computed on the basis of the amount of mortar in the mix, rather than on the total volume of the concrete. Figure 5 plots strength ratio versus fiber spacing. Here the spacing was computed on the basis of the mortar content when all aggregate retained on a No. 4 sieve was excluded from the fiber volume percentage calculation. The spacing at which the fibers become effective is reduced several thousands for both the ¾ in. and ⅜ in. mixes. The curves of Figures 4 and 5 were drawn from data obtained by the method of least squares. For purposes of curve fitting, it was assumed that the fibers were ineffective at a spacing of 0.40 in. This corresponds to a volume percentage of 0.35.

It is difficult to explain the inability of the fibers to reinforce the mortar. However, Williamson,[14] working with a 1:2 mortar mix and 2.0 volume percent fiber, reported a 5 percent reduction in compressive strength for 4000 psi concrete, using 4 × 8

11G. R. Williamson, *Some Static and Dynamic Characteristics of Fiber-Reinforced Concrete*, Ph.D. Dissertation, Carnegie Mellon University (May 1969).

12T. T. C. Hsu, "Mathematical Analysis of Shrinkage Stresses in a Model of Hardened Concrete," *ACI Journal Proceedings*, Vol 60 (March 1963).

13G. R. Williamson, *Some Static and Dynamic Characteristics of Fiber Reinforced Concrete*, Ph.D. Dissertation, Carnegie Mellon University (May 1969).

14G. R. Williamson, *Some Static and Dynamic Characteristics of Fiber Reinforced Concrete*, Ph.D. Dissertation, Carnegie Mellon University (May 1969).

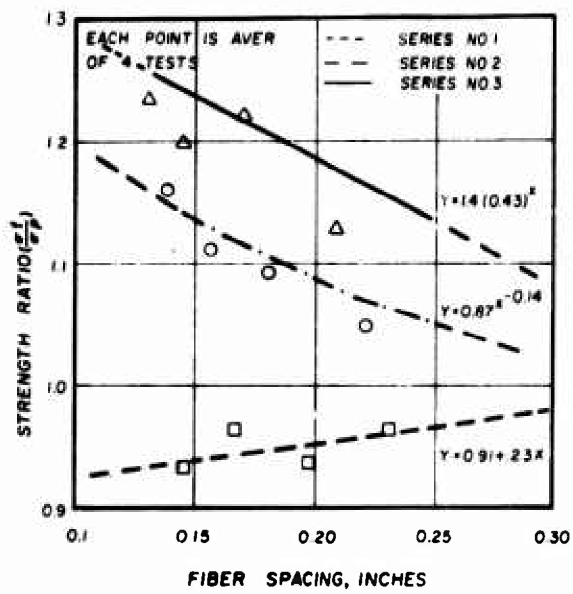

Figure 5. Strength ratio at a function of fiber spacing computed for mortar content only (6 x 12 in. cylinders).

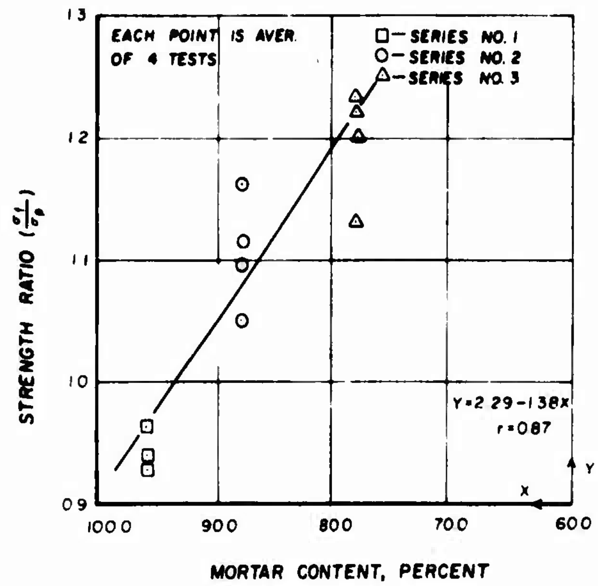

Figure 6. Strength ratio versus mortar content for each of the three mixes (6 x 12 in. cylinders).

in. cylinders. Hsu and Slate[15] found that the tensile bond strength between the mortar and coarse aggregate decreased with increasing amounts of sand. The bond strength developed for a 1:2 mortar was 15 to 20 percent greater than for 1:3 mortar. This was attributed to the increased tensile stresses at the paste aggregate interface due to the volume change of the paste. Hsu[15] has shown that these tensile stresses vary inversely as the distance between aggregate particles. The estimated clear distance between sand particles for 1:3 mortar was reported to be only one-fourth that for 1:2 mortar. The influence of the sand content upon the tensile bond developed between the paste and aggregate (and therefore between the paste and the fiber) could account for the widely scattered results reported for the effect of the fibers on the compressive strength of fiber-reinforced concrete. A plot of the strength ratio versus the percentage of mortar in each of the three mixes is shown in Figure 6. The data (the least squares fit) show the variation of the strength-ratio with mortar content to be linear. The curve shows that a strength ratio less than 1.0 can be expected for a mortar content in excess of 93 percent.

[15] T. T. C. Hsu and F. O. Slate, "Tensile Bond Strength Between Aggregate and Cement Paste or Mortar," *ACI Journal, Proceedings*, Vol 60, No. 4 (April 1963).

[16] T. T. C. Hsu, "Mathematical Analysis of Shrinkage Stresses in a Model of Hardened Concrete," *ACI Journal, Proceedings*, Vol 60, No. 3 (March 1963).

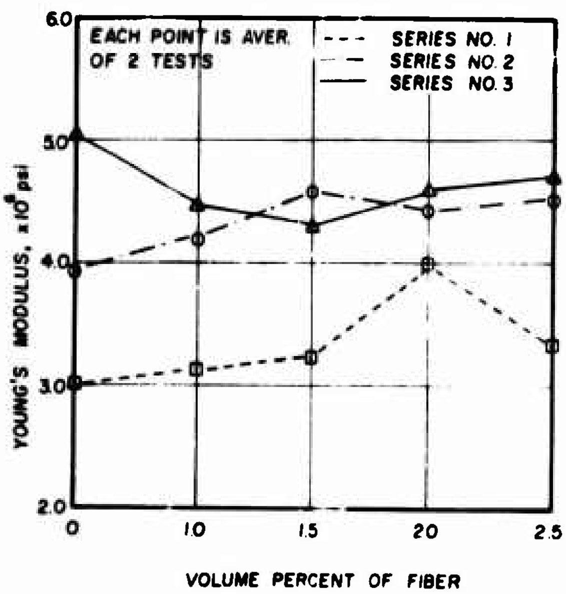

Figure 7. Young's modulus as a function of fiber content (6 x 12 in. cylinders).

Modulus of Elasticity and Poisson's Ratio. Figure 7 is a plot of the secant modulus versus volume percent of fiber. The moduli were computed at 50 percent of ultimate. For Series No. 1 and Series No. 2 there is a slight increase in modulus as the amount of fiber increases. There is a slight decrease in the modulus with the addition of the fibers although Series No. 3 is not affected by the volume

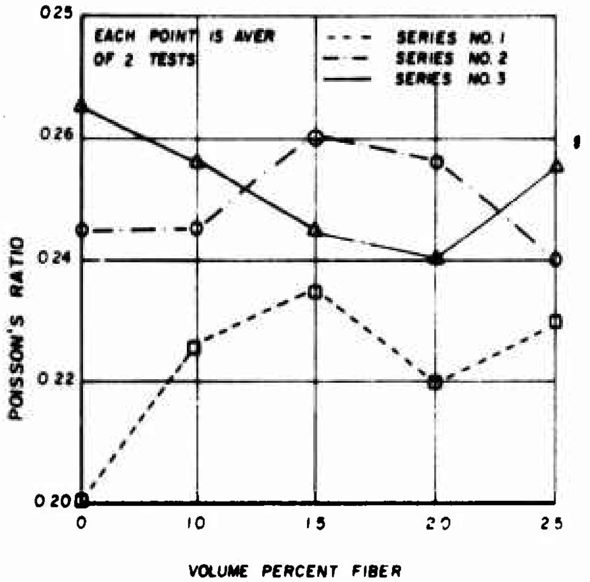

Figure 8. Poisson's Ratio as a function of fiber content. (6 x 12 in. cylinders).

percent of fiber. The modulus of the mortar mix is roughly 75 percent of the modulus of the two concrete mixes. This is probably due to the influence of the coarse aggregate and lower water-cement ratios of the concretes.

Figure 8 shows Poisson's ratio as a function of fiber content. Again, for Series Nos. 1 and 2, the fibers tend to increase Poisson's ratio, while for Series No. 3, the fibers tend to decrease it. For the plain mixes, Poisson's ratio for the mortar is about 80 percent of that for the concretes. However, for the fiber mixes, the value for the mortar is within 92 percent of that for the concretes.

Affect of Cylinder Size on Ultimate Strength. Figure 9 is a plot of the compressive strength with respect to the volume percent of fiber for each of the three cylinder sizes for each series. The 6 × 12 in. cylinder strengths averaged 4 percent less than the 4 × 8 in. and 3 × 6 in. cylinders; the greatest difference occurred with the plain mixes. This difference is similar to that reported by the Bureau of Reclamation.[17] The curves of Figure 9 give no indication that the Weibull effect need be considered for the three sizes of cylinders used in this study.

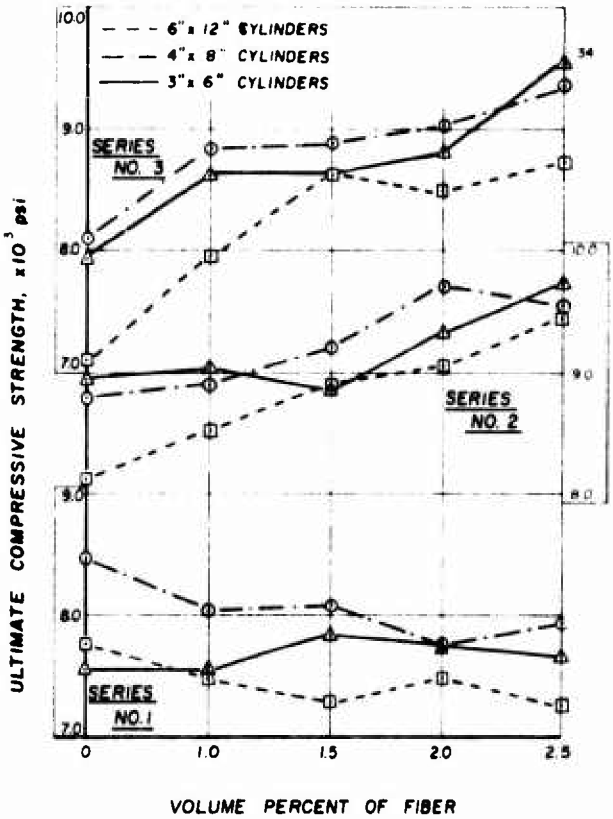

Figure 9. The effect of cylinder size on the ultimate compressive strength.

Ductility. Figures 10 through 12 are stress-strain curves for each of the three test series. Each point is the average of two tests. (The individual curves are found in Appendix D.) These figures show that the addition of fibers has little or no effect on the ductility of either mortar or concrete. These results are not unexpected, however, since the testing was performed on a "soft" testing machine, that is, one which stores large amounts of energy during loading of the specimen and then releases this energy at the onset of yielding of the specimen. If the energy released by the machine is greater than that required for further compression of the specimen, fracture occurs rather abruptly. If, however, the energy released is less than that required for further compression of the specimen, then additional load must be applied by the machine in order to cause failure. This allows further recording of the load-deformation curve and a more accurate indication of the ductility of the material.

[17] *Concrete Manual* (U.S. Department of the Interior, Bureau of Reclamation, 5th Edition, 1951).

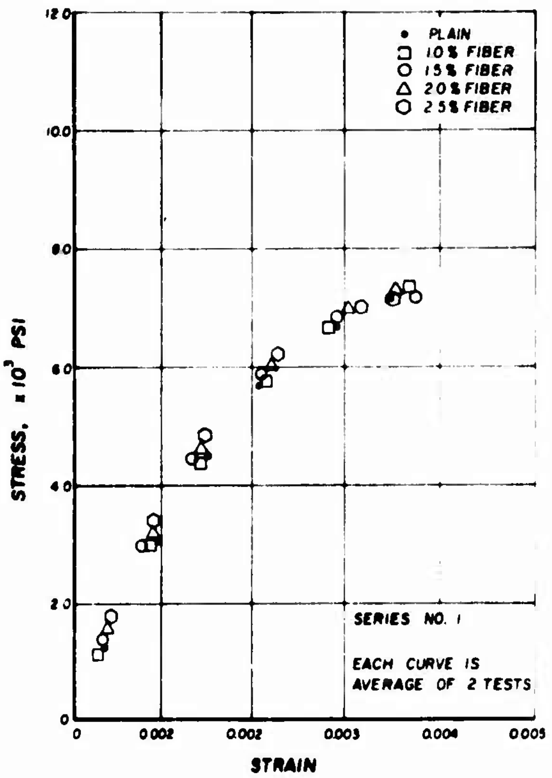

Figure 10. Stress-strain curves for mortar.

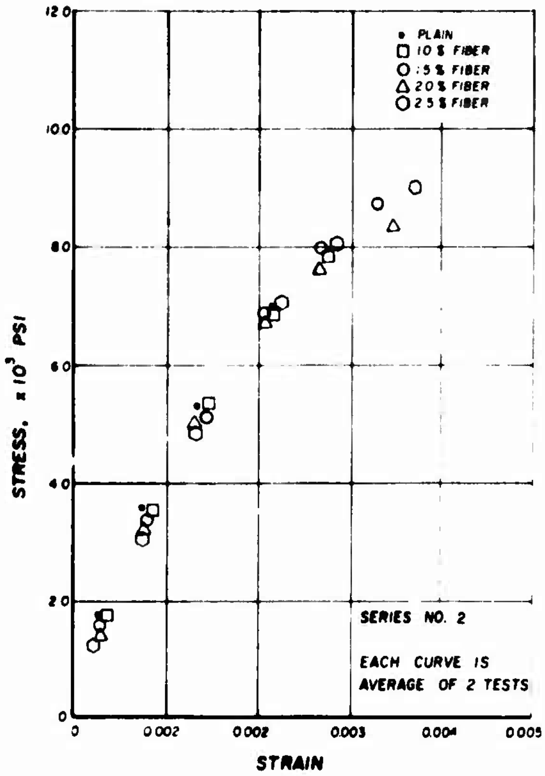

Figure 11. Stress-strain curves for 3/8 in. maximum aggregate concrete.

4 APPLICATIONS

General. Results of this study are most useful in the design of reinforced concrete flexural members where compressive strength, shear strength, and ductility are important design parameters. As previously mentioned, the shear strength and ductility of fiber-reinforced concrete have been studied by others. Batson et al.,[18] using 4 in. × 6 in. × 6 ft beams without conventional shear reinforcement, found that moment failures could be induced with as little as 0.44 volume percent fiber. Williamson[19] showed the high ductility of a fiber-reinforced beam with as little as 1.0 volume percent fiber. Assuming that these results are applicable to full-scale beams, these data, and the data of this report, are the basis for a cost comparison of a conventionally reinforced beam with stirrups as shear reinforcement, and the same beam with fibers instead of stirrups. The designs, using the Strength Design Method as given in American Concrete Institute (ACI) 318-71, are found in Appendix C.

Design Criteria. The beams are designed for a live load of 2.0 kips per ft on a simple span of 24 ft, with 5,000 psi concrete and ¾ in. maximum aggregate. One volume percent of fiber by volume of concrete is used in the fibrous beam. Figure 1 indicates that this permits a 10 percent increase in the compressive strength of the concrete. This fiber percentage is well in excess of that which Batson[20] reported as necessary to assure a moment failure.

The Strength Design Method is based on the

[18] G. B. Batson, E. Jenkins, and R. Spatney, "Steel Fibers as Shear Reinforcement in Beams," *ACI Journal Proceedings*, Vol 69, No. 10 (October 1972).

[19] G. R. Williamson, *Fibrous Reinforcement for Portland Cement Concrete*, Technical Report No. 2-40 (Ohio River Division Laboratories [ORDL], May 1965).

[20] G. B. Batson, E. Jenkins, and R. Spatney, "Steel Fibers as Shear Reinforcement in Beams," *ACI Journal, Proceedings*, Vol 69, No. 10 (October 1972).

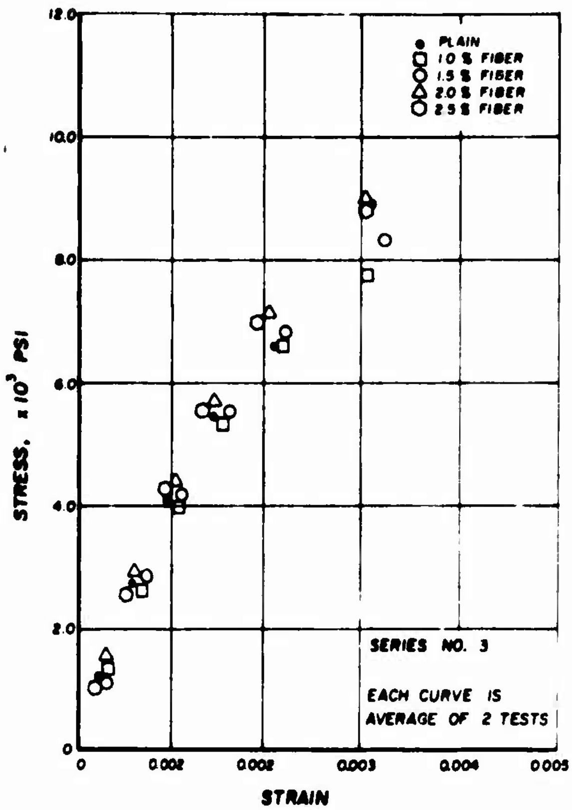

Figure 12. Stress-strain curves for ¾ in. maximum aggregate concrete.

assumption that failure of the concrete in compression occurs at a strain of 0.003. This value is taken from the original plastic design (ultimate strength design) work of Whitney.[21] Whitney recognized that although concrete cylinders failed in compression at a strain under 0.002, the compressive strain at failure in a beam was in the vicinity of 0.004, or twice that of the cylinders. He recommended that 0.003 be used for Plastic Design procedures. Because the ductility of the concrete is increased in bending by the addition of the fibers,[22] and because the fibers prevent catastrophic failure of concrete in compression,[23,24] it is reasonable to increase the assumed strain of the concrete in beams at failure. Therefore, the failure strain of the fibrous beam in this study was taken as 0.0033, an increase of 10 percent.

In the design of the fiber beam, the fibers below the neutral axis were assumed to be effective in flexure. Schrader,[25] using this assumption, achieved excellent correlation between theoretical and experimental values for the resisting moment of fiber reinforced beams. His calculations were based upon a bond force of 10 lbs for each fiber assumed effective in the longitudinal direction. The actual load per fiber was taken as 0.70 of this value.

The unit costs used to compare the two designs (see Appendix C) are prevailing costs (Jan 73) for the Pittsburgh, Pennsylvania, area.

As stated above, the beam designs are in accordance with the Building Code Requirements of ACI 318-71. However, the deflection of the beams and the cutting off of the main reinforcement were not considered since these refinements would be approximately equal for both beams and would not affect the cost comparison.

Results and Discussion. The cost of the beams as calculated in Appendix C shows that cost of the beam with stirrups is $16 greater than the fibrous beams ($267 to $251). The regular concrete beam is 1 in. deeper and has 40 percent more main reinforcing steel.

For beam in torsion, the omni-directional effectiveness of the fibers could possibly permit the elimination of the torsional reinforcement, which also usually takes the form of stirrups. This could increase the competitive position of flexural members made with steel fibers even more.

5 CONCLUSIONS AND RECOMMENDATIONS

1. The addition of steel fibers to concrete significantly increases compressive strength.
2. The addition of steel fibers to mortar may decrease compressive strength.
3. Steel fibers do not significantly affect Young's modulus or Poisson's ratio for concrete, but do show a slight tendency to increase these parameters for mortar.
4. There is a strong indication of an inverse relationship between the compressive strength of

[21] C. S. Whitney, "Plastic Theory in Reinforced Concrete Design," *Transactions ASCE*, Vol 107 (1942).

[22] G. R. Williams, *Fibrous Reinforcement for Portland Cement Concrete*, Technical Report No. 2-40 (Ohio River Division Laboratories [ORDL], May 1965).

[23] *Ibid.*

[24] D. L. Birkimer and J. R. Hossley, *Comparison of Static and Dynamic Behavior of Plain and Fibrous Reinforced Concrete Cylinders*, Technical Report No. 4-69 (ORDL, January 1968).

[25] E. K. Schrader, *Studies in the Behavior of Fiber-Reinforced Concrete*, Master's Thesis, Clarkson College of Technology (April 1971).

fiber concrete and the mortar content of the mix.

5. The 6 × 12 in. cylinders had up to 4 percent less compressive strength than the 4 × 8 in. and 3 × 6 in. cylinders.
6. The use of fibers in reinforced concrete beams is economical.

As fibrous concrete becomes practical and economical, the applications will increase greatly. To meet this need the most effective design procedures are required. Research such as this study not only contributes to the development of design methods, but also points out where information or knowledge is lacking and suggests areas where additional research should be conducted. Results of this study indicate the following research is needed:

1. Full-scale tests to determine the flexural behavior of reinforced concrete beams that contain steel fibers in place of conventional shear and torsion reinforcement.
2. A program to determine the validity of the Beam Design Theory presented here.
3. A detailed study to determine the effect of mortar content on the compressive strength of steel fiber concrete.
4. A program to determine the effect of steel fibers on Young's modulus and Poisson's ratio for mortar.

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Schrader, E. K., *Studies in the Behavior of Fiber-Reinforced Concrete*, Master's Thesis, Clarkson College of Technology (April 1971).

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Williamson, G. R. and B. H. Gray, *Technical Information Pamphlet on Use of Fibrous Concrete* (Construction Engineering Research Laboratory [CERL], May 1973).

APPENDIX A: ASTM SPECIFICATIONS

The following American Society for Testing Materials specifications were followed in the preparation and testing of the specimens used in this study:

1. Making and Curing Concrete Test Specimens in the Laboratory, C192-68.
2. Compressive Strength of Molded Concrete Cylinders, C39-66.
3. Sieve or Screen Analysis of Fine and Coarse Aggregate, C136-67.

APPENDIX B: INDIVIDUAL TEST RESULTS

Table B1

Test Series No. 1

Basic Mix — 1:3:W/C = 0.61. Max. Agg. = No. 4 Sieve. Slump = 6".
Age at Test — 28 days. Water Cure — 18 days. Air Dry — 10 days.

6" x 12" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation	E 10^6 psi	μ
17-1A-31	0	7010				2.77	0.19
17-1A-32	0	7360	7780	605.0	0.078		
18-1A-31	0	8460				3.27	0.21
18-1A-32	0	8260					
17-1B-31	1.0	7010				2.83	0.21
17-1B-32	1.0	7360	7470	396.0	0.053		
18-1B-31	1.0	8100				3.48	0.24
18-1B-32	1.0	7380					
17-1C-31	1.5	7020					
17-1C-32	1.5	7170	7290	202.0	0.028	3.11	0.22
18-1C-31	1.5	7560				3.41	0.25
18-1C-32	1.5	7350					
17-1D-31	2.0	7110					
17-1D-32	2.0	7050	7470	393.0	0.053	3.68	0.22
18-1D-31	2.0	7740				4.28	0.22
18-1D-32	2.0	7960					
17-1E-31	2.5	6820				3.23	0.25
17-1E-32	2.5	6760	7275	484.0	0.067		
18-1E-31	2.5	7710				3.44	0.21
18-1E-32	2.5	7800					

4" x 8" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation
17-1A-21 •	0	8460			
17-1A-22	0	8200	8470	189.0	0.022
18-1A-21	0	8640			
18-1A-22	0	8680			
17-1B-21	1.0	8060			
17-1B-22	1.0	8060	8040	251.0	0.031
18-1B-21	1.0	8390			
18-1B-22	1.0	7680			
17-1C-21	1.5	7740			
17-1C-22	1.5	7880	8080	327.0	0.040
18-1C-21	1.5	8110			
18-1C-22	1.5	8600			
17-1D-21	2.0	7160			
17-1D-22	2.0	7650	7750	213.0	0.027
18-1D-21	2.0	7560			
18-1D-22	2.0	8050			
17-1E-21	2.5	8070			
17-1E-22	2.5	7750	7950	151.0	0.019
18-1E-21	2.5	7880			
18-1E-22	2.5	8130			

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Table B1 (Cont)

3" x 6" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation
17-1A-11	0	7190			
17-1A-12	0	7520	7530	282.0	0.037
18-1A-11	0	7880			
18-1A-12	0	—			
17-1B-11	1.0	7750			
17-1B-12	1.0	7580	7540	208.0	0.028
18-1B-11	1.0	7190			
18-1B-12	1.0	7610			
17-1C-11	1.5	6620			
17-1C-12	1.5	7590	7710	724.0	0.094
18-1C-11	1.5	8600			
18-1C-12	1.5	8030			
17-1D-11	2.0	7590			
17-1D-12	2.0	7190	7740	357.0	0.046
18-1D-11	2.0	8060			
18-1D-12	2.0	8030			
17-1E-11	2.5	7330			
17-1E-12	2.5	7250	7595	308.0	0.041
18-1E-11	2.5	7940			
18-1E-12	2.5	7860			

Table B2

Test Series No. 2

Basic Mix -- 1:1.44:096:W/C = 0.55. Max. Agg. = 3/8". Slump = 9".
Age at Test — 28 days. Water Cure — 18 days. Air Dry — 10 days.

6" x 12" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation	E 10^6 psi	μ
26-2A-31	0	7960				3.90	0.25
26-2A-32	0	8100	8130	251.0	0.031		
27-2A-31	0	7890				3.98	0.24
27-2A-32	0	8580					
26-2B-31	1.0	8450				3.51	0.25
26-2B-32	1.0	9270	8530	597.0	0.07		
27-2B-31	1.0	7625				4.66	0.24
27-2B-32	1.0	8760					
26-2C-31	1.5	8510				4.68	0.25
26-2C-32	1.5	8740	8900	352.0	0.040		
27-2C-31	1.5	8890				4.48	0.27
27-2C-32	1.5	9465					
26-2D-31	2.0	8225				4.52	0.27
26-2D-32	2.0	9555	9050	651.0	0.072		
27-2D-31	2.0	8435				4.34	0.24
27-2D-32	2.0	9685					
26-2E-31	2.5	9025				4.57	0.25
26-2E-32	2.5	9590	9440	288.0	0.030		
27-2E-31	2.5	9360				4.48	0.23
27-2E-32	2.5	9800					

Table B2 (Cont)

4" x 8" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation
26-2A-21	0	8805			
26-2A-22	0	8880	8790	165.0	0.019
27-2A-21	0	8520			
27-2A-22	0	8955			
26-2B-21	1.0	8835			
26-2B-22	1.0	8935	8895	72.0	0.008
27-2B-21	1.0	8995			
27-2B-22	1.0	8820			
26-2C-21	1.5	8955			
26-2C-22	1.5	9035	9195	210.0	0.023
27-2C-21	1.5	9315			
27-2C-22	1.5	9475			
26-2D-21	2.0	9675			
26-2D-22	2.0	9755	9685	134.0	0.014
27-2D-21	2.0	9475			
27-2D-22	2.0	9835			
26-2E-21	2.5	9235			
26-2E-21	2.5	9395	9535	234.0	0.025
27-2E-21	2.5	9835			
27-2E-22	2.5	9675			

3" x 6" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation
26-2A-11	0	9045			
26-2A-12	0	8745	8945	133.0	0.015
27-2A-11	0	9085			
27-2A-12	0	8905			
26-2B-11	1.0	8805			
26-2B-12	1.0	9045	9030	192.0	0.021
27-2B-11	1.0	8945			
27-2B-12	1.0	9330			
26-2C-11	1.5	8535			
26-2C-12	1.5	9250	8845	311.0	0.035
27-2C-11	1.5	8550			
27-2C-12	1.5	9045			
26-2D-11	2.0	—			
26-2D-12	2.0	9720	9320		
27-2D-11	2.0	9620			
27-2D-12	2.0	8550			
26-2E-11	2.5	9695			
26-2E-12	2.5	10235	9800	613.0	0.062
27-2E-11	2.5	8845			
27-2E-12	2.5	10425			

Table B3

Test Series No. 3

Basic Mix — 1:1.44:0.96: W/C (02 Series) — 0.54. Slump = 4" W/C (03 Series) = 05
Slump = 8". Age at Test — 28 days. Water Cure — 18 days. Air Dry — 10 days.

6" x 12" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation	E 10^6 psi	μ
02-3A-31	0	8220	(7965)	255.0	0.032	5.08	0.28
02-3A-32	0	7710	7070				
03-3A-31	0	5760	(6165)	405.0	0.066	4.96	0.25
03-3A-32	0	6570					
02-3B-31	1.0	7940	(8330)	390.0	0.047	4.82	0.25
02-3B-32	1.0	8720	7970				
03-3B-31	1.0	7580	(7590)				
03-3B-32	1.0	7600					
02-3C-31	1.5	8960	(9385)	425.0	0.045	4.51	0.22
02-3C-32	1.5	9810	8620				
03-3C-31	1.5	7550	(7840)	290.0	0.037	4.16	0.27
03-3C-32	1.5	8130					
02-3D-31	2.0	10060	(9670)	390.0	0.040	4.78	0.25
02-3D-32	2.0	9280	8490			4.42	0.23
03-3D-31	2.0	7300	(7330)	30.0	0.004		
03-3D-32	2.0	7360					
02-3E-31	2.5	9060	(9640)	580.0	0.060	4.78	0.24
02-3E-32	2.5	10220	8710				
03-3E-31	2.5	7880	(7795)	85.0	0.011	4.65	0.27
03-3E-32	2.5	7710					

4" x 8" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation
02-3A-21	0	9060	(9060)	0.0	0.0
02-3A-22	0	9060	8100		
03-3A-21	0	7080	(7120)	40.0	0.006
03-3A-22	0	7160			
02-3B-21	1.0	9350	(9350)	0.0	0.0
02-3B-22	1.0	9350	8820		
03-3B-21	1.0	8060	(8280)	220.0	0.027
03-3B-22	1.0	8500			
02-3C-21	1.5	9850	(9765)	135.0	0.014
02-3C-22	1.5	9580	8870		
03-3C-21	1.5	7920	(8015)	95.0	0.012
03-3C-22	1.5	8110			
02-3D-21	2.0	10330	(10070)	260.0	0.026
02-3D-22	2.0	9810	9010		
03-3D-21	2.0	7860	(7925)	65.0	0.008
03-3D-22	2.0	7990			
02-3E-21	2.5	10520	(10030)	490.0	0.049
02-3E-22	2.5	9540	9350		
03-3E-21	2.5	8460	(8640)	180.0	0.021
03-3E-22	2.5	8820			

Table B3 (Cont)

3" x 6" Cylinders

Specimen Number	Fiber %	Ult. Comp. Stress, psi	Average Stress, psi	Standard Deviation	Coefficient of Variation
02-3A-11	0	8180	(8425)		
02-3A-12	0	8670	7960	506.0	0.064
03-3A-11	0	7370	(7490)		
03-3A-12	0	7610			
02-3B-11	1.0	8750	(911[illegible])		
02-3B-12	1.0	[illegible]	86[illegible]	581.0	0.068
03-3B-11	1.0	9[illegible]	([illegible]95)		
03-3B-12	1.0	8240			
02-3C-11	1.5	9830	(8965)		
02-3C-12	1.5	8100	8615	715.0	0.083
03-3C-11	1.5	8100	(8260)		
03-3C-12	1.5	8420			
02-3D-11	2.0	8260	(9090)		
02-3D-12	2.0	9920	8800	720.0	0.080
03-3D-11	2.0	8150	(8505)		
03-3D-12	2.0	8860			
02-3E-11	2.5	10290	(10305)		
02-3E-12	2.5	10420	9535	841.0	0.088
03-3E-11	2.5	8430	(8730)		
03-3E-12	2.5	9030			

Table B4

Verification Test for Test Series No. 1

Basic Mix — 1:3:W/C = 0.61. Max. Agg. = No. 4 Sieve
Age at Test — 28 days. Water Cure — 18 days. Air Dry — 10 days.

4" x 8" Cylinders

Specimen Number*	Ultimate Strength, psi Vibrated	Rodded
31-1A-21	8630	8315
31-1A-22	8590	8470
31-1A-23	8550	8350
31-1A-24	8630	8510
31-1B-21	8710	8470
31-1B-22	8750	8510
31-1B-23	8750	8270
31-1B-24	8710	—
31-1D-21	8730	8315
31-1D-22	8510	8470
31-1D-23	8710	8510
31-1D-24	8670	8545

*Each number represents two cylinders; one compacted by rodding and one compacted by use of a table vibrator.

A Plain Mortar
B 1.0% Fiber
C 2.0% Fiber

APPENDIX C: BEAM DESIGNS AND COST ANALYSIS

The symbols listed here are for ease in interpreting the following section.*

Symbols

f'_c — ultimate compressive strength of concrete

f_y — yield stress of reinforcing steel

M_u — ultimate design moment

M_t — theoretical ultimate design moment

ρ — reinforcing ratio = AS/bd

ρ_b — reinforcing ratio for balanced conditions

b — width of beam

d — depth of beam to centroid of steel

A_s — area of tension reinforcement

z — quantity limiting distribution of flexural reinforcement

d_c — thickness of concrete cover

v_c — allowable shear stress in concrete

v_u — ultimate shear stress

A_v — area of shear reinforcement

s — spacing of stirrups

*See ACI 318-71 for details.

Design of Rectangular Beam with Only Tension Reinforcement

Given: $f'_c = 5000$ psi

$f_y = 50{,}000$ psi

Service Load: Live Load = 2.0 k/ft
Dead Load = Assume 0.225 k/ft (12 × 18 in.)

Span: 24 ft — 0 in. center to center, simply supported

Columns: 12 × 12 in.

Design Load: $U = 1.4D + 1.7L = (1.4)(.225) + 1.7\,(2.0)$

$$U = 0.31 + 3.40 = 3.71 \text{ k/ft}$$

$$M_u = \frac{wl^2}{8} = \frac{3.71\,(24)^2}{8} = 267 \text{ k/ft} = 3200 \text{ k/in.}$$

$$M_t = \frac{M_u}{\varphi} = \frac{3200}{.9} = 3560 \text{ k/in.}$$

For Balanced Design:

$$\varrho_b = 0.85\,\beta_1 \frac{f'_c}{f_y}\left(\frac{87000}{87000 + f_y}\right)$$

$$= 0.85\,(.80)\left(\frac{5000}{50000}\right)\left(\frac{8700}{8700 + 50000}\right) = 0.0432$$

$$\varrho_{max} = .75\,\varrho_b = .75\,(.0432) = .0324$$

$$M_t = db^2 f'_c\,\omega(1 \quad 0.59\omega) \text{ where } \omega = \frac{\varrho f_y}{f'_c}$$

$$= .0324\left(\frac{50000}{50000}\right)$$

$$= .0324$$

Assume b = 12 in.

$$3{,}560{,}000 = 12\,d^2\,5000\,(0.324)\,[1 - .59\,(.324)]$$

$d = 15.02$ in. $\simeq 15.0$ in.

For Computational Purposes Only, Assume #10 bars

Total Depth of Beam: 15 in. + ½ bar dia + ⅝ in.

Stirrups + 1½ in. Cover: D = 15 + ⅝ in. + ⅝ in.

+ 1½ in. = 17½ in.

Assumed Depth: 18 in. > 17.5 in. Ok

$$A_s = \varrho_b d = .0324\,(12)\,(15) = 5.83 \text{ sq in.}$$

$d_c = 2.5$ in.

$$A = 2\,\frac{d_c b}{\text{No.}} = \frac{2(2.5)(12)}{5} = 12$$

$$f_s = 0.6\,f_y = 0.6\,(50) = 30$$

$$z = 30\,\sqrt[3]{2.5\,(12.0)} = 93.3 < 175.0 \quad \text{Ok}$$

Design of Shear Reinforcement

At Face of Supports $V_u = 3.71\,\frac{(24.0 - 1.0)}{2} = 42.65$ k

@ d from Support $V_u = 42.65 - 3.71\,(\frac{15}{12}) = 38.0$ k

Assume L.L. Fixed, $V_c = 0$

$$v_c = 2\sqrt{f'_c} = 2\sqrt{5000} = 141 \text{ psi}$$

$$v_u = \frac{V_u}{\varphi bd} = \frac{38000}{0.85(12)\,(15)} = 249 \text{ psi}$$

x = Dist. stirrups no longer needed

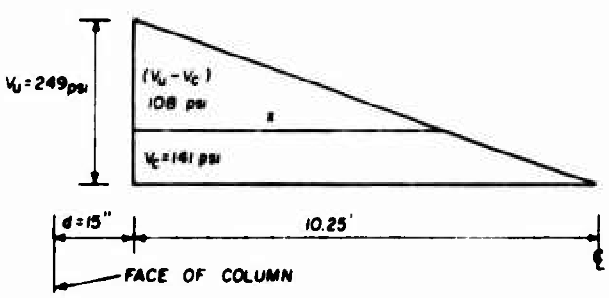


Figure C-1. Design of shear reinforcement

$$\frac{x}{10.25} = \frac{108}{249}$$

$x = 4.45$ ft

Total dist. stirrups required $= x + 2d = 4.45 + 2(1.25)$

$= 6.95$ ft $\simeq 84$ in.

Assume #3 stirrups: $A_v = 0.22$ sq in.

$f_y = 40000$ psi

$$A_v = (v_u - v_c)\frac{b_w s}{f_y}$$

$$0.22 = (249 - 141)\frac{(12)\,s}{40000}$$

$s = 6.8$ in.

$s_{max} = d/2 = \frac{15}{2} = 7.5$ in. ← Controls

$$s_{max} = \frac{A_u f_y}{50 b_w} = \frac{.22\,(40000)}{50(12)} = 14.6 \text{ in.}$$

@ $x' = 2.25'$ $\quad v_u - v_c = 195 - 141 = 54$ psi

$$s_x' = \frac{(.22)(40000)}{54\,(12)} = 13.6 \text{ in.}$$

Use 6 stirrups @ 6½ in. = 32½ in. + d = 47½ in.

Use 5 stirrups @ 7½ in. = $\frac{37\tfrac{1}{2}\text{ in.}}{85 \text{ in.} > 84}$ required Ok

It is customary to run some stirrups full length of beam.

Use 4 @ 12 in. = 48 in.

2 - #6 bars 24 ft-0 in. long will be placed in top of beam to tie stirrups.

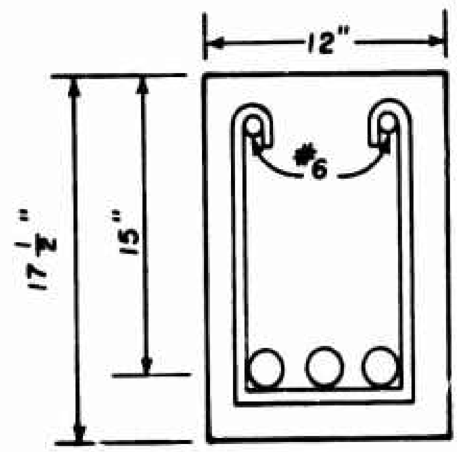


Figure C-3. Use of bars to tie stirrups.

Design of Beam with Fibers and Tension Reinforcement

Given: $f'_c = 5500$ psi

$f_y = 50000$ psi

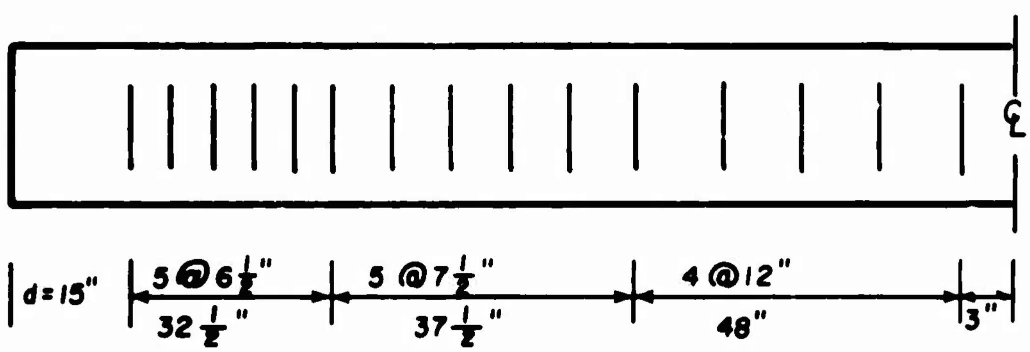


Figure C-2. Placement of stirrups on beam.

Service Load: Live Load 2.0 k/ft

Dead Load Assume 12 × 16½ in. beam = .206 k/ft

Span: 24 ft-0 in. C to C, Simply Supported

Columns: 12 × 12 in.

Design Load: U = 1.4 (.206) + 1.7 (2.0) = 3.69 k/ft

$$M_u = \frac{wl^2}{8} = \frac{3.69\,(24)^2}{8} = 266 \text{ k/ft}$$

$$M_t = \frac{266}{.9} = 296 \text{ k/ft} = 3550 \text{ k-in.}$$

For Balanced Design:

$$\varrho_{max} = .03798 = .038$$

$$M_t = bd^2 f'_c\, \omega\,(1 - .59\omega) \qquad \omega = .038\,(\frac{50}{5.5}) = .346$$

Assume b = 12 in. $\qquad .59\,(0.346) = 0.204$

$3550 = 12d^2\, 5.5\,(0.346)\,(1 - 0.204)$

$$d^2 = \frac{3550}{18.2} = 195.0$$

d = 14.0 in.

Total Depth: D = 14 + ½ bar dia. + ⅜ in. stirrups + 1½ in. cover

D = 16.5 in. = 16.5 in. assumed OK

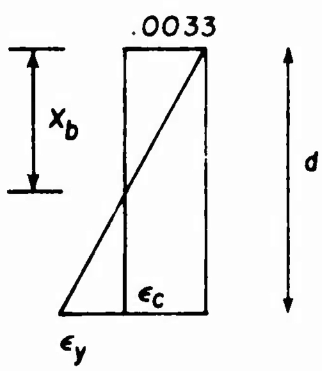


Figure C-4. Design of beam with fibers and tension reinforcement.

$$\frac{X_b}{d} = \frac{.0033}{.0033 + f_y}$$

$$X_b = \frac{.0033}{.0033 + f_y} \cdot d = \frac{.0033}{.0033 + \frac{f_y}{29 \times 10^6}} \cdot d$$

$$X_b = \frac{95700}{95700 + f_y} \cdot d$$

$$C_b = 0.85\, f'_c\, bk_1\, X_b$$

$$T_b = A_{sb} f_y = \varrho_b bd\, f_y$$

$$C_b = 0.85\, f'_c\, bk_1\, (\frac{95700}{95700 + f_y}) \cdot d = \varrho_b bd\, f_y$$

$$C_b = 0.85\, \beta_1 \frac{f'_c}{f_y}\, (\frac{95700}{95700 + f_y})$$

Assume 10 percent increase in f'_c due to fibers:

$f'_c = 5500;\ \beta_1 = 0.825$

$$\varrho_b = 0.85\,(.825)\,\frac{5.5}{50}\,(\frac{95.7}{95.7 + 50}) = .05064$$

$$\varrho_{max} = .75\,\varrho_b = .75\,(.5064) = .03798$$

For d = 14.5 in.

$$X_b = \frac{95700}{95700 + 50000} \cdot (14.5) = 9.55 \text{ in.}$$

Calculation of Resisting Moment of Fibers on Tension Side of Beam

For 1 percent volume percentage of 0.010 × 0.022 × 1.0 in. fiber (equivalent diameter = d_f = 0.017 in.). Number of fibers effective in any one direction per square inch of area = n.

$$n\pi \frac{d_f^2}{4}\, l = \text{Vol}$$

Using l = 1 in. Volume = 0.01 cu in. and 0.41 effective in any direction

$$n = 4\,\frac{(.01)(.41)}{3.14(.017)^2} = 18.8 \text{ Fibers/sq in.}$$

According to the work of Schrader* the force required to pull out one fiber is 10 lbs. The average force in all the fibers from the extreme tension fiber to the neutral axis is $0.7 \times 10 = 7$ lbs. Concentrating this force at the center of the tension area and neglecting the small displacement of the neutral axis:

$$T_f = 18.8\,(7.)\,(12)\,(6.95) = 11{,}000 \text{ lbs.}$$

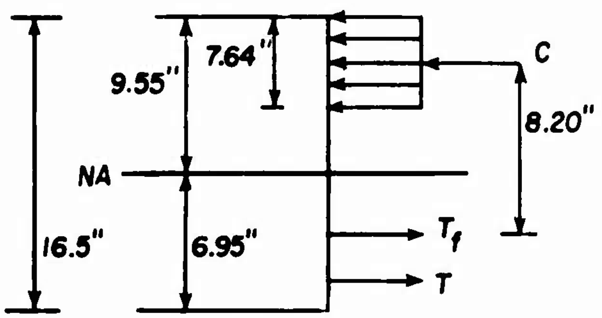


Figure C-5. Resisting force of fibers.

$a = .8\,X_b = (.8)(9.55)$

$a = 7.64$ in.

Cost of Beam Without Fibers

Approximate length of stirrup: 42 in.

Weight per stirrup: $\frac{42}{12}(0.376) = 1.31$ lbs

Weight of stirrups per beam: $1.31\,(28) = 36.7$ lb

Quantity of Concrete: $1.0 \times \frac{17.5}{12} \times 24 = 35.0$ cu ft

Quantity of Forms: $(2 \times \frac{17.5}{12} + \frac{12}{12}) \times 24 = 94.0$ sq ft

Quantity of Main Rein.: $\frac{5.83}{144}(24) = 0.973$ cu ft $= 476$ lbs

Concrete—Material $\frac{35.0}{27}$ (24.0)	\$ 31.10
Placing $\frac{35.0}{37}$ (4.80)	6.22
Forms—Material 94 (0.20)	18.80
Labor 94 (1.20)	112.70
Reinforcing—Material 476 (.0945)	45.00
Placing 476 (.0730)	34.70
Stirrups—Material 36.7 (.0965)	3.54
Placing 36.7 (.0730)	2.68
Bars to hold stirrups—Material	6.82
Placing	5.26
	\$266.82
Total Cost of Beam Without Fibers	\$267.00

Cost of Beam With 1 Percent Fiber

Quantity of Concrete: $\frac{12}{12} \times \frac{16.5}{12} \times 24 = 33$ cu ft

Quantity of Forms: $(2 \times \frac{16.5}{12} + \frac{12}{12}) \times 24 = 90$ sq ft

Quantity of Reinforcing: $\frac{4.16}{144}(24) = 0.695$ cu ft $= 341.0$ lbs

Quantity of Fiber: $(.01)(33) = 0.33$ cu ft $= 162.0$ lbs

Concrete—Material $\frac{33.0}{27}$ (24.0)	\$ 29.40
Placing $\frac{33.0}{27}$ (4.80)	5.86
Forms—Material 90 (0.20)	18.00
Labor 90 (1.20)	108.00
Reinforcing—Material 341 (.0945)	32.20
Placing 341 (.0730)	24.90
Fibers—Material 162 (0.18)	29.10
Placing* 162 (0.02)	3.24
	250.70
Total Cost of Beam with Fibers	\$251.00

*E. K. Schrader. *Studies in the Behavior of Fiber-Reinforced Concrete.* Master's Thesis, Clarkson College of Technology (April 1971).

*Two cents per pound is assumed to cover the cost of batching and handling of the fibers.

$M_f = 11{,}000\ (8.2) = 90300$ in/lbs $= 45.15$ k/in.

Moment to be resisted by main reinforcement:*

$M' = M_t \quad M_f = 266.0 \quad 45.1 = 220.9$ k/in.

$$K_u = \frac{M'}{bd^2} = \frac{220.9\,(12000)}{12\,(14.5)^2} = 1048\ \#/\text{sq in}$$

$$\varrho = \frac{1}{m}\,(1\sqrt{1 \quad 2\,\frac{mk_u}{f_y}})$$

$$\text{where } m = \frac{f_y}{.85f'_c} = \frac{50000}{.85\,(5500)} = 10.67$$

$$\varrho = \frac{1}{10.67}\,(1\sqrt{1 \quad \frac{2\,(10.67\,(1048))}{50000}})$$

$\varrho = 0.0239$

$A_s = \varrho bd = 0.0239\,(12)\,(14.5) = 4.16$ sq in.

Cost Analysis

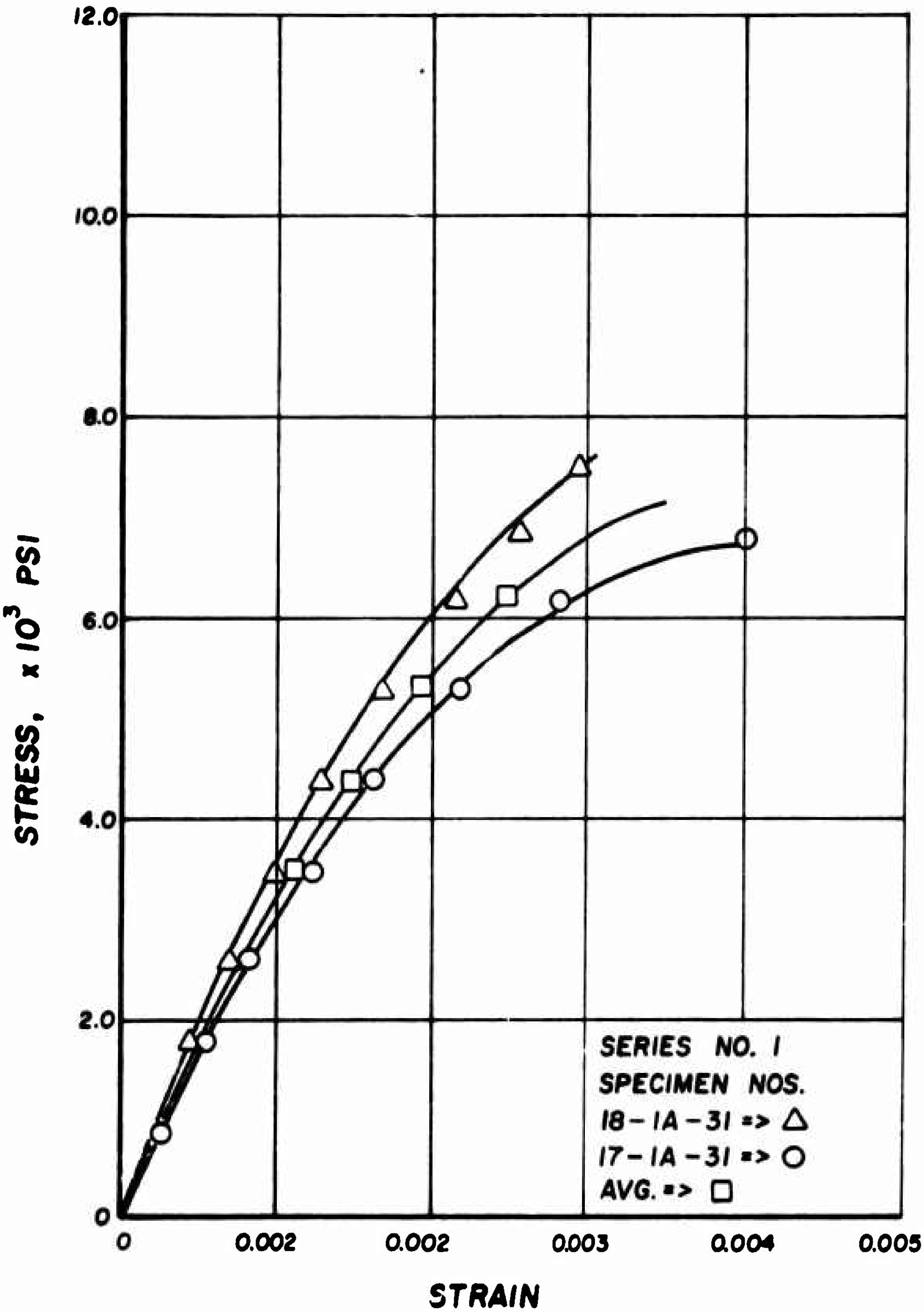
The following cost comparison of the beam without fibers with the beam with fibers is based upon the prevailing (Jan 73) costs for the Pittsburgh, Pennsylvania, area.

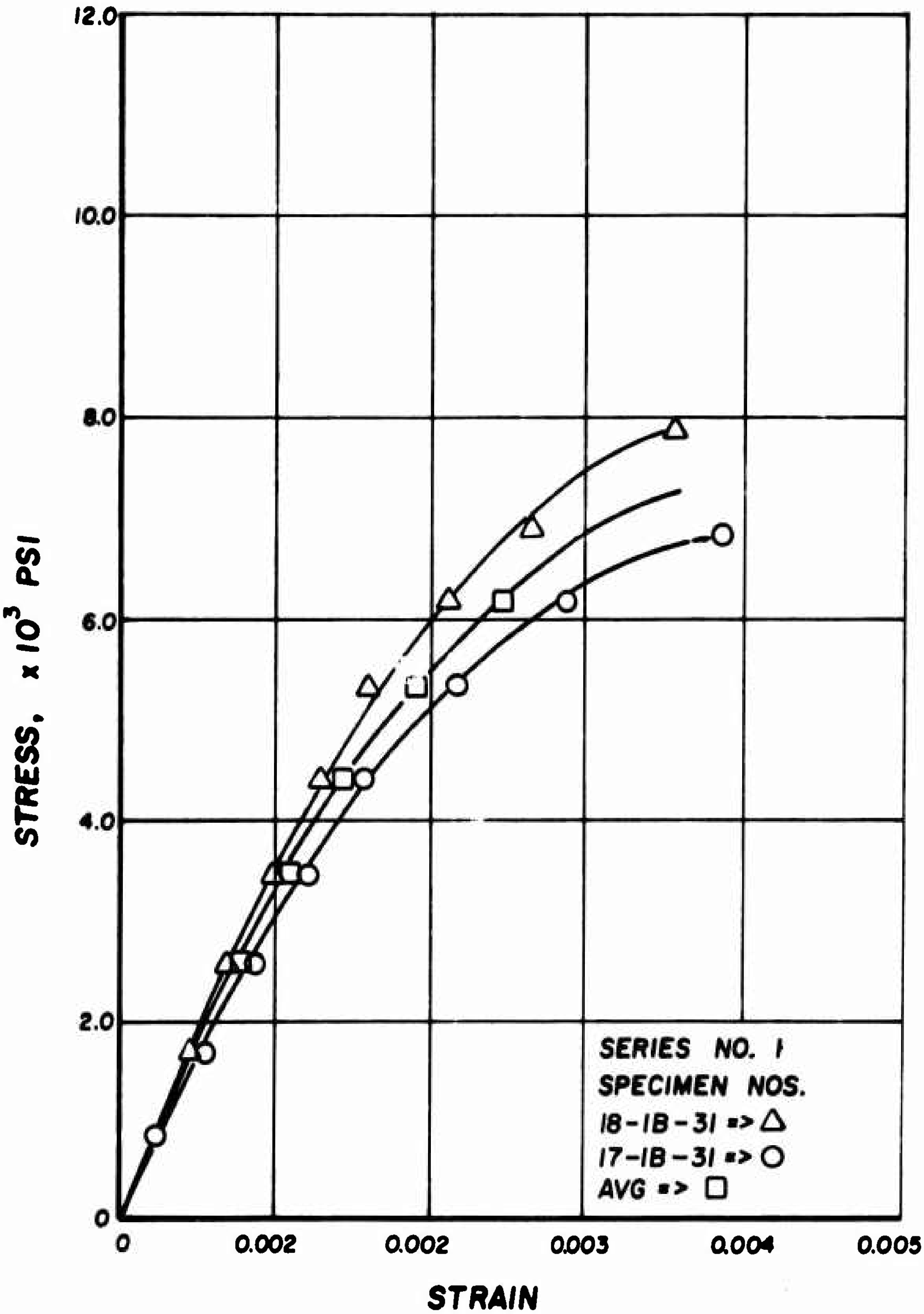
5000 psi Concrete—Material		\$ 24.00 cu yd
	Placing	4.80 cu yd
Stirrups—Material		0.0965 per lb
	Placing	0.0730 per lb
Main Reinforcing—Material		0.0945 per lb
	Placing	0.0730 per lb
Form Work—Material		0.0200 sq ft
	Labor	1.2000 sq ft

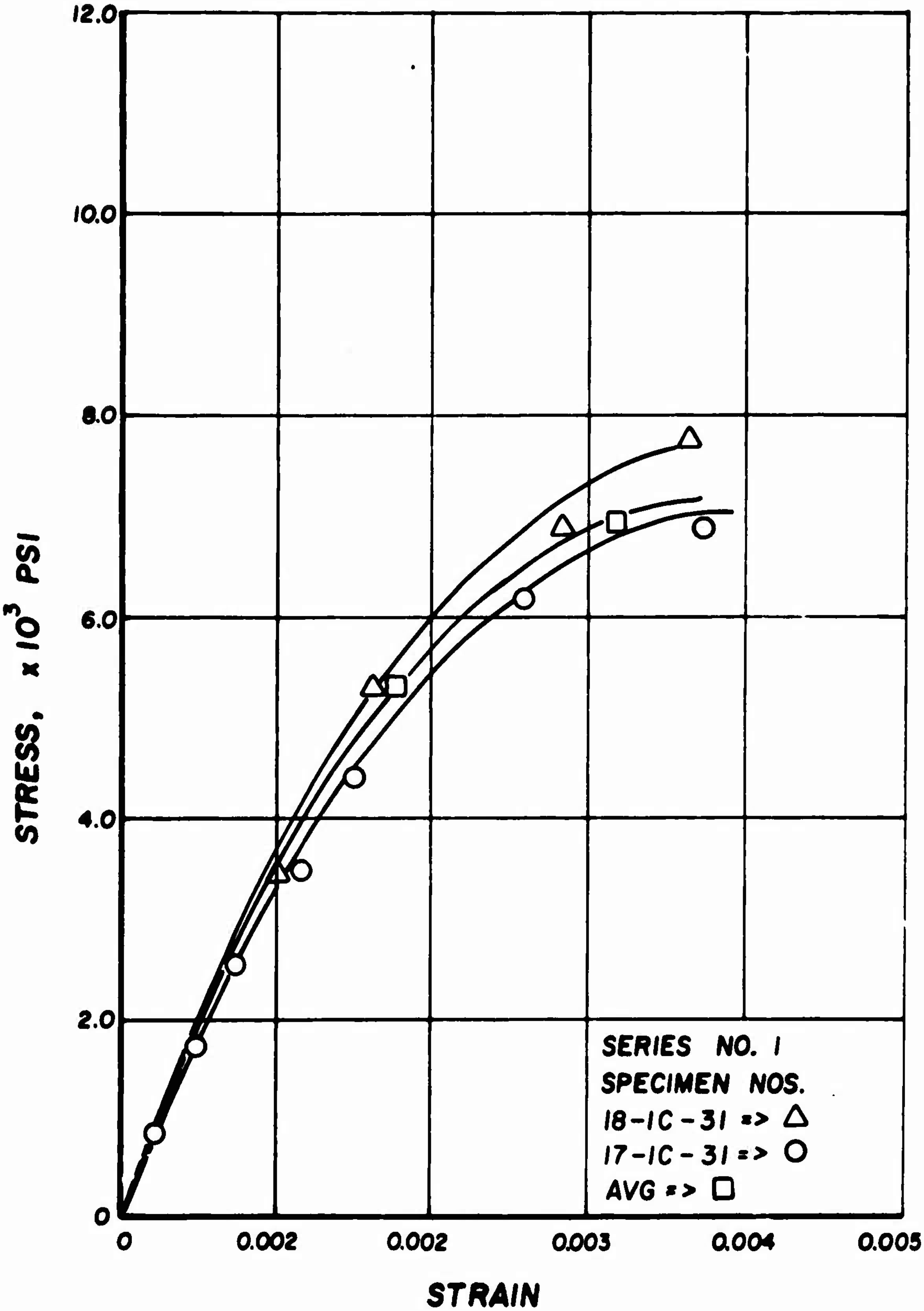
The following labor and material costs for the Pittsburgh, Pennsylvania, area as of January 1973.

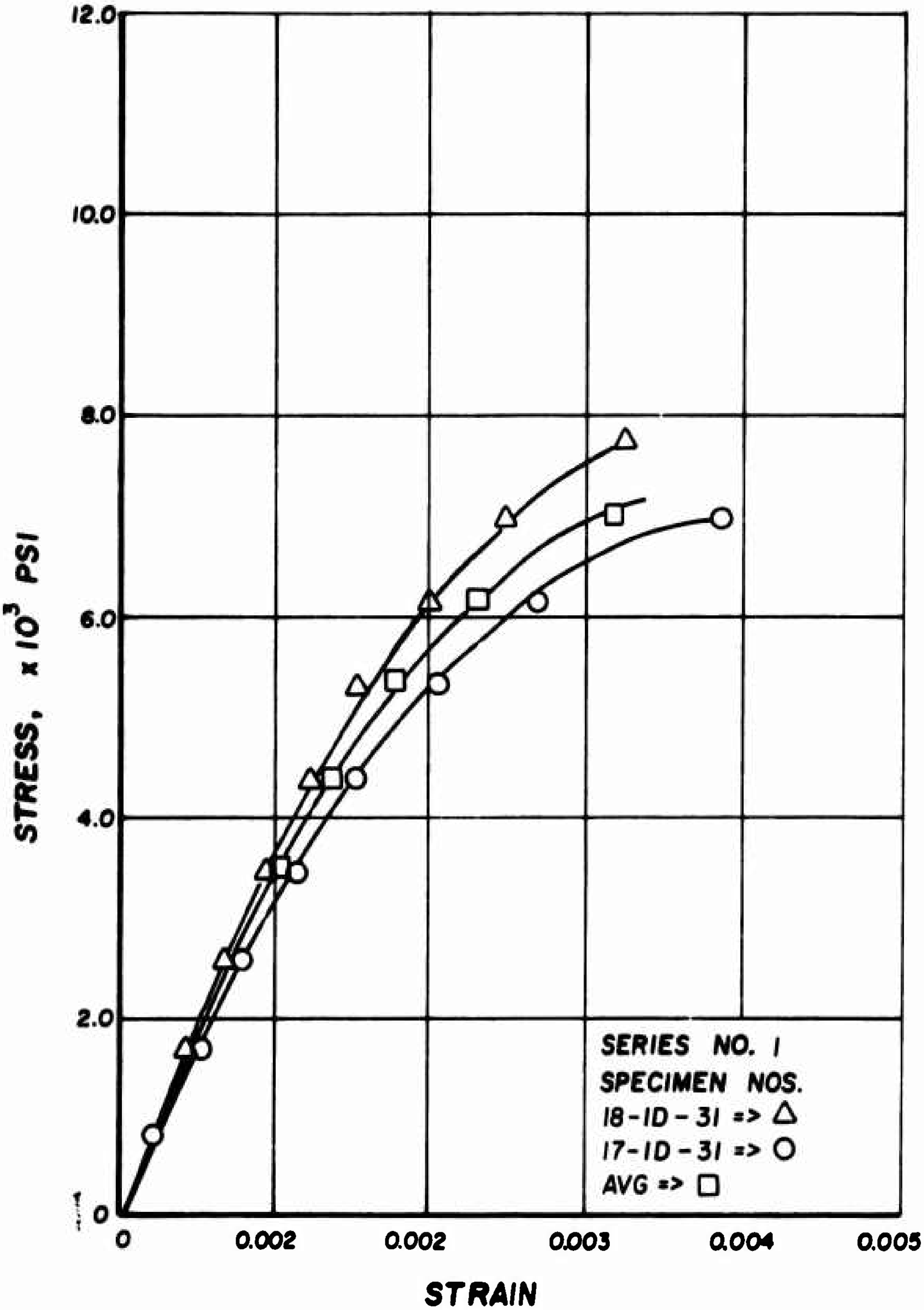
5000 psi Concrete—Material		\$ 24.00 cu yd
	Placing	4.86 cu yd
Stirrups—Material		193.00 per ton
	Placing	146.00 per ton
Main Reinforcing—Material		189.00 per ton
	Placing	146.00 per ton
Form Work—Material		0.20 sq ft
	Labor	1.20 sq ft

APPENDIX D: TYPICAL STRESS-STRAIN CURVES FOR COMPRESSION TESTS SHOWN IN TABLES B1, B2 AND B3

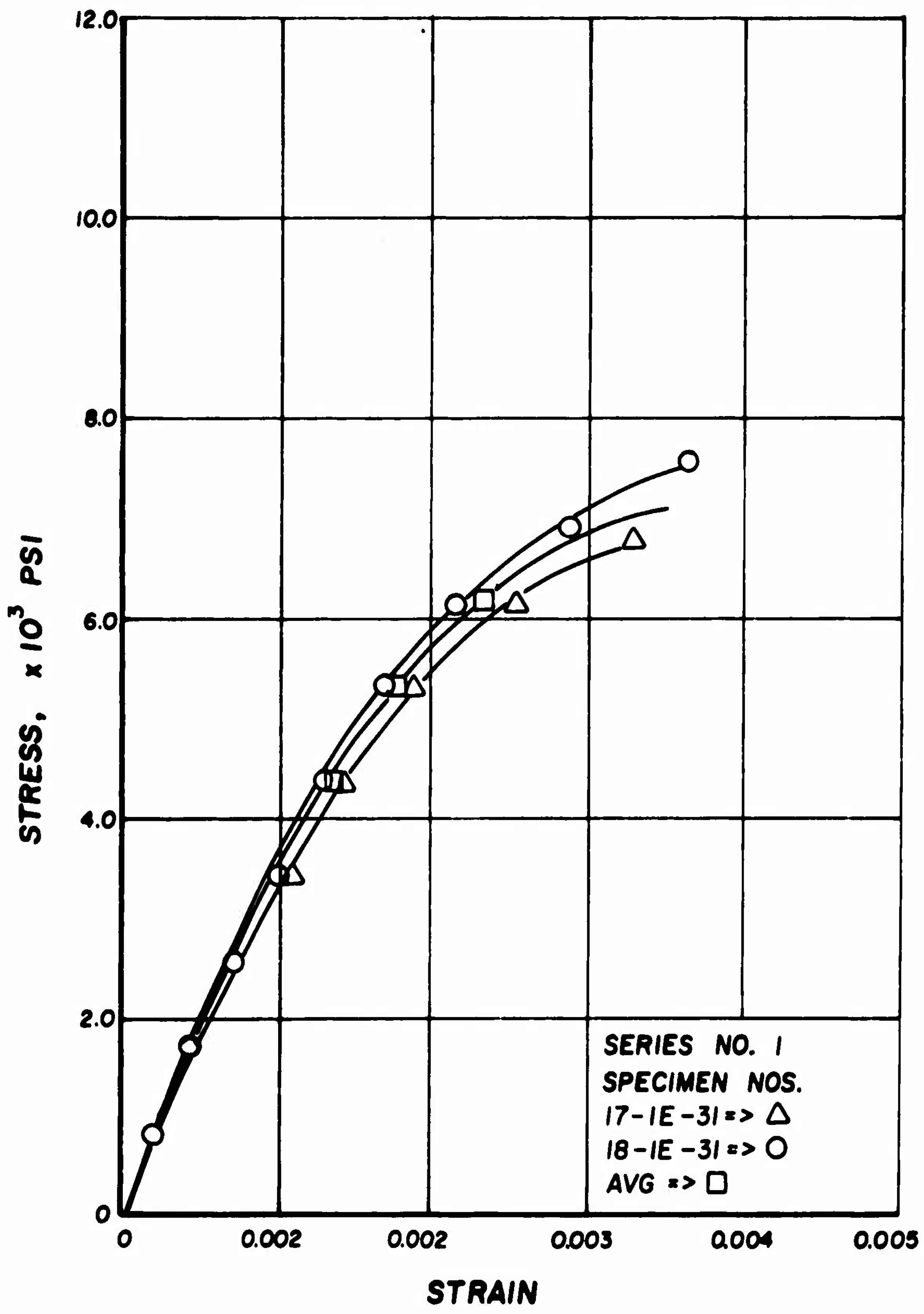

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 1
SPECIMEN NOS.
18 - 1A - 31 => △
17 - 1A - 31 => ○
AVG. => □

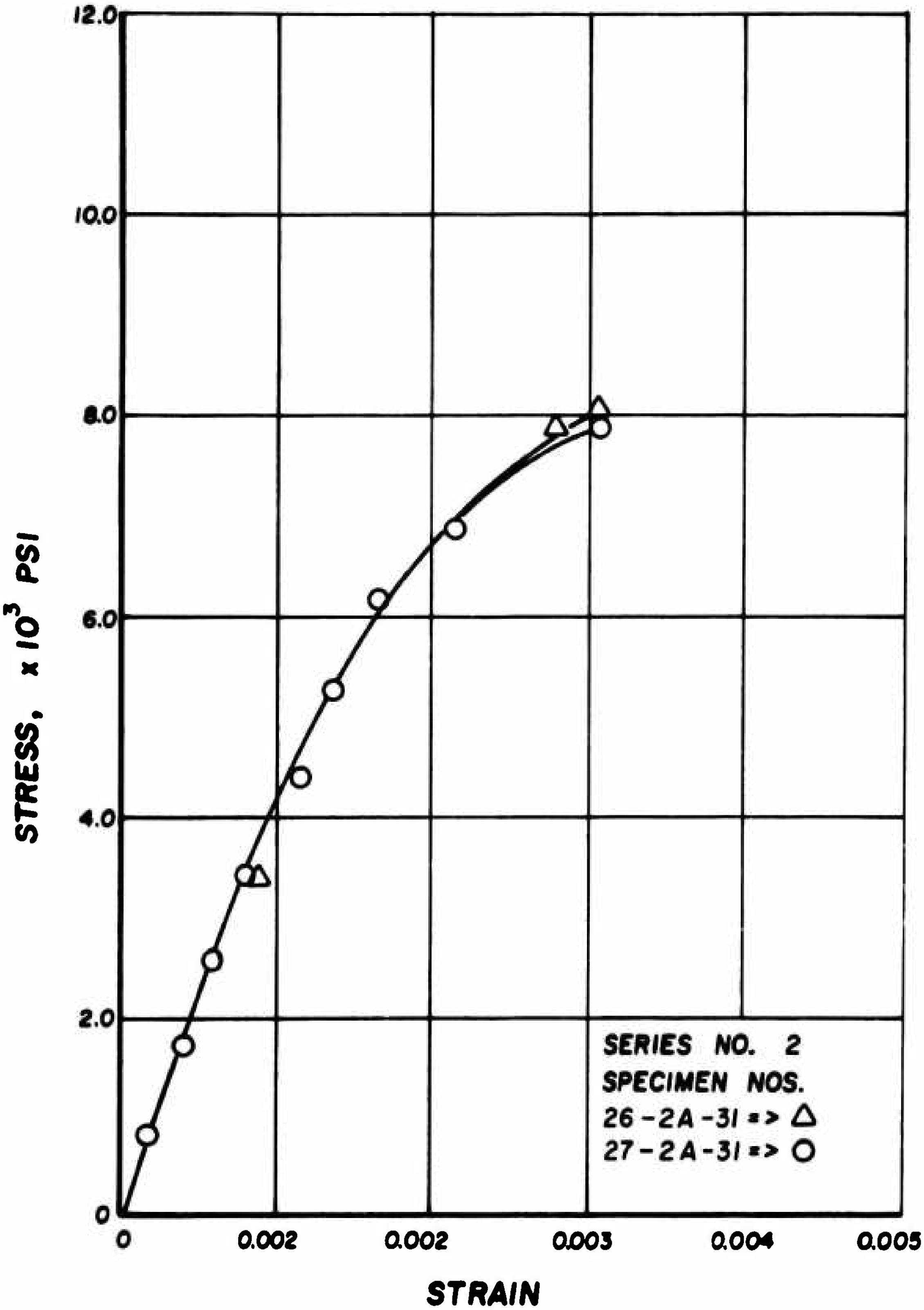

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 1
SPECIMEN NOS.
18-1B-31 => △
17-1B-31 => ○
AVG => □

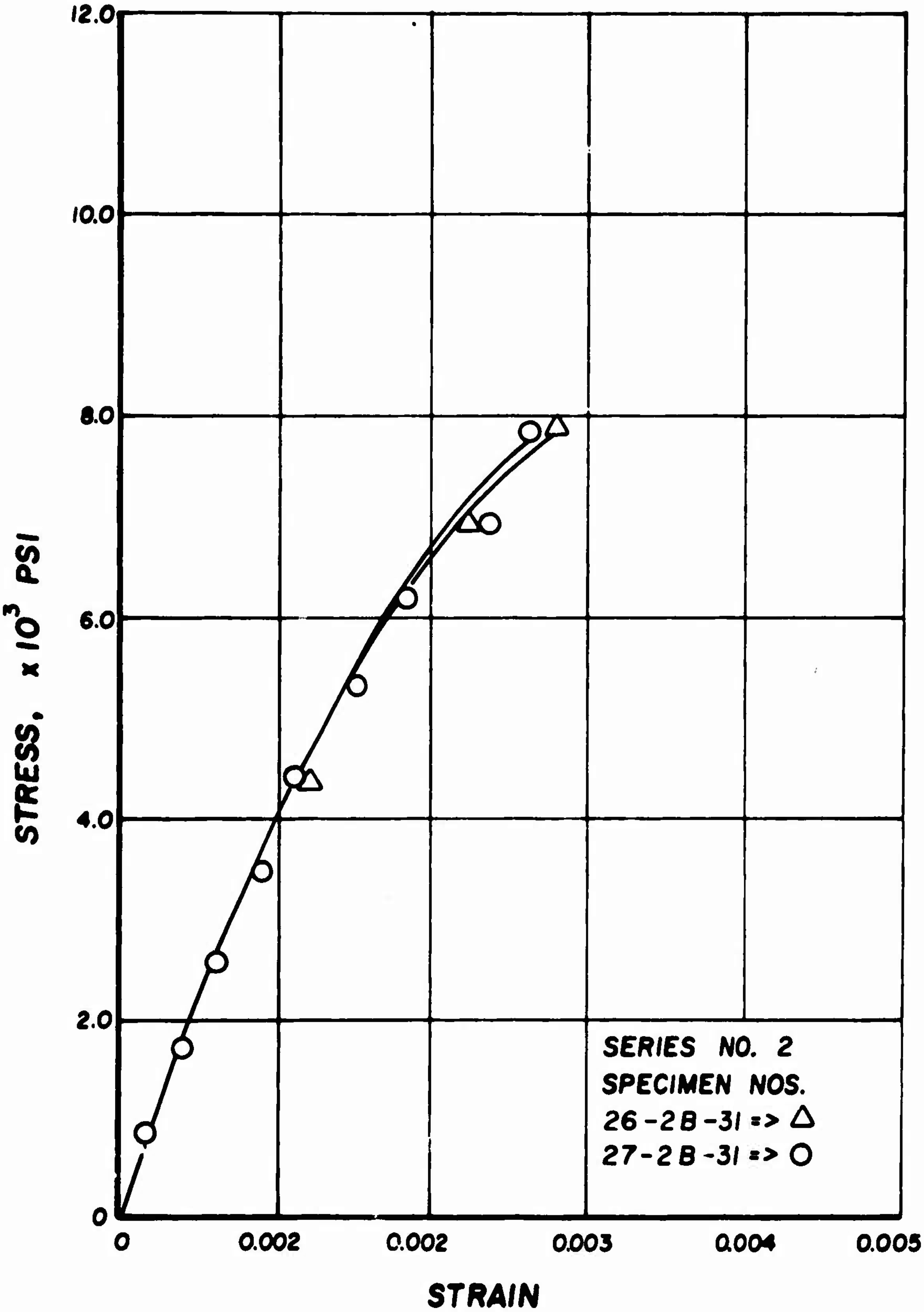

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. I
SPECIMEN NOS.
18-IC-31 => △
17-IC-31 => ○
AVG => □

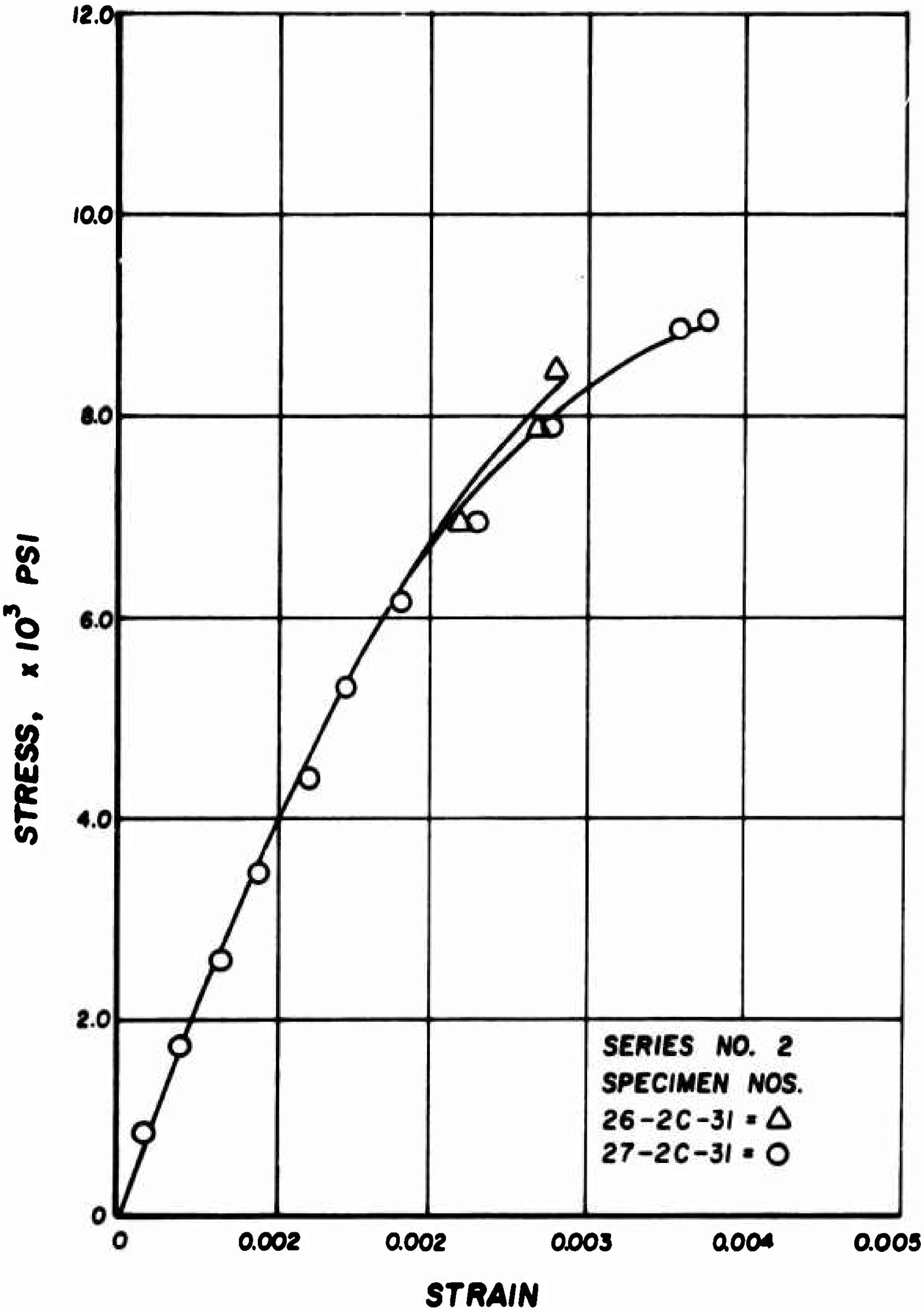


12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10³ PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 1
SPECIMEN NOS.
18-1D-31 => △
17-1D-31 => ○
AVG => □

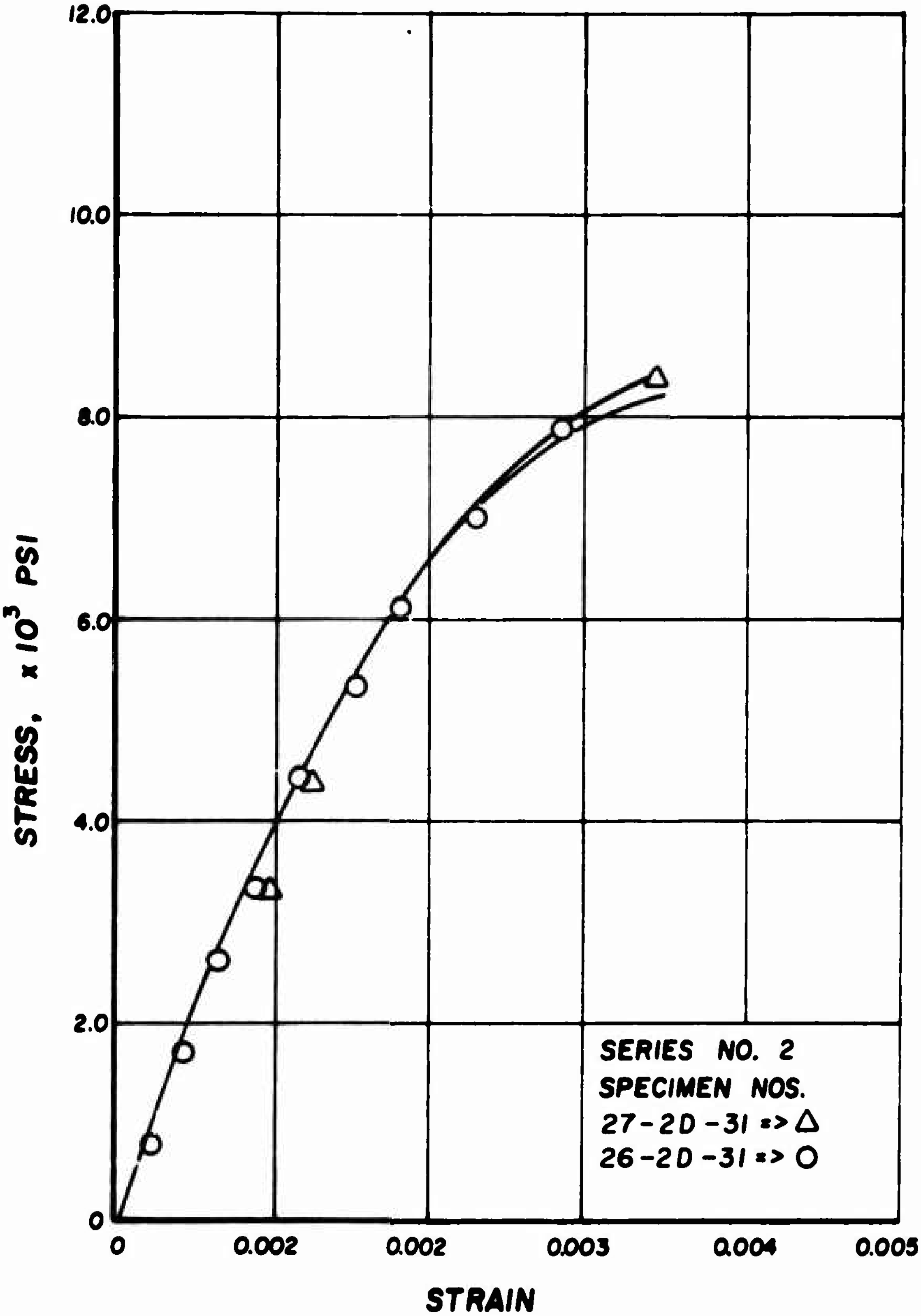

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10³ PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 1
SPECIMEN NOS.
17-1E-31 => △
18-1E-31 => ○
AVG => □

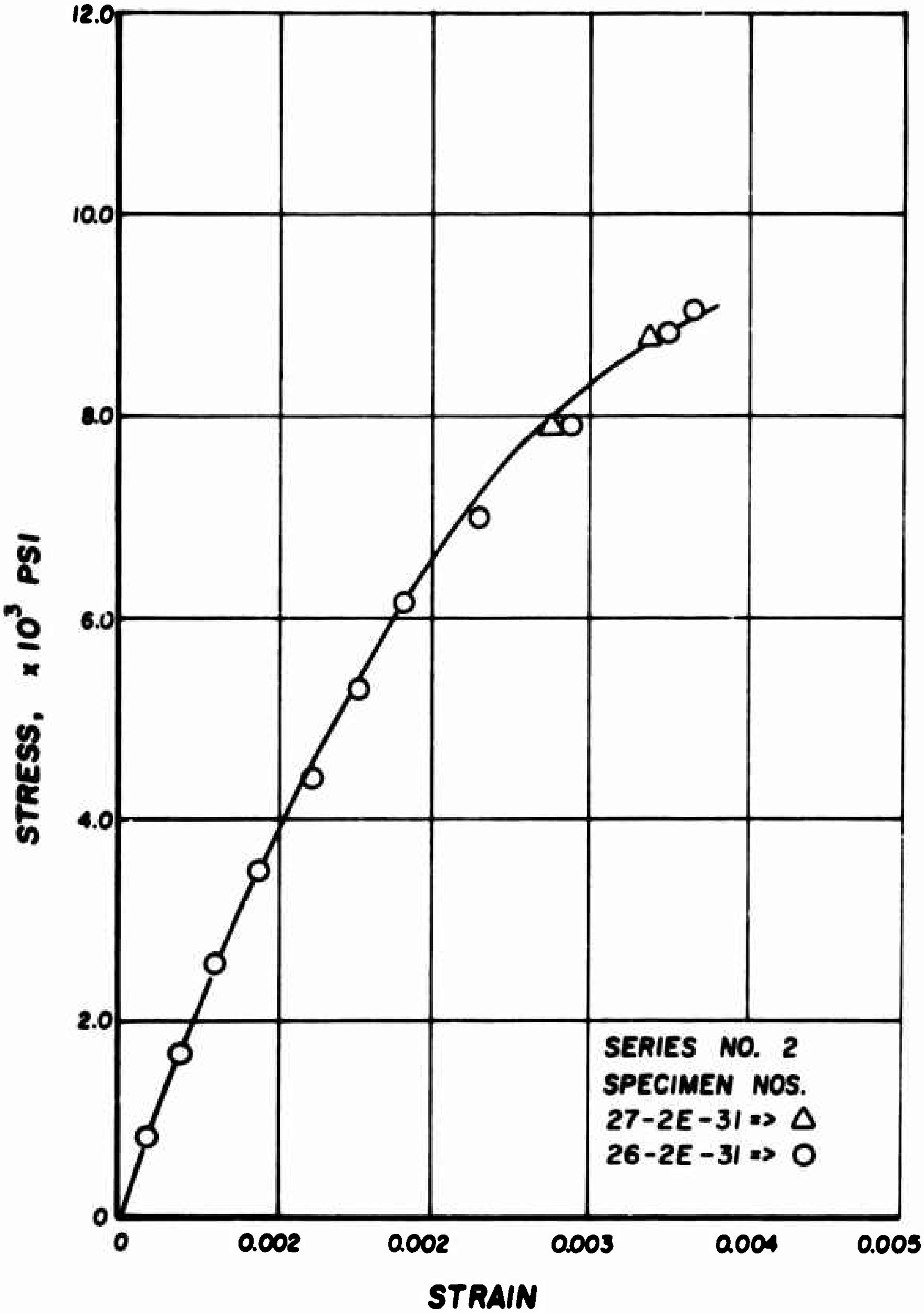

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 2
SPECIMEN NOS.
26-2A-31 => △
27-2A-31 => ○

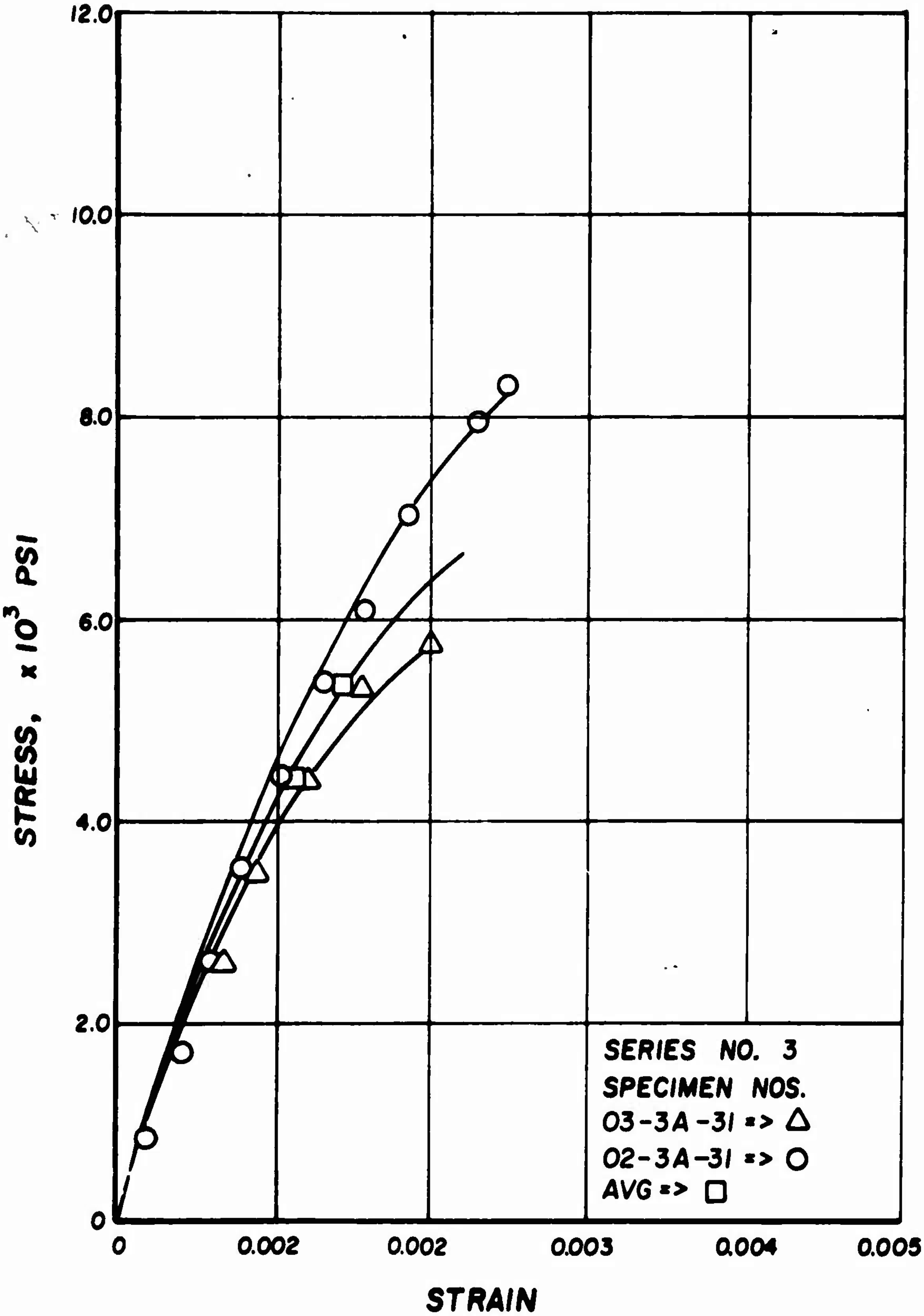

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10³ PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 2
SPECIMEN NOS.
26-2B-31 => △
27-2B-31 => O

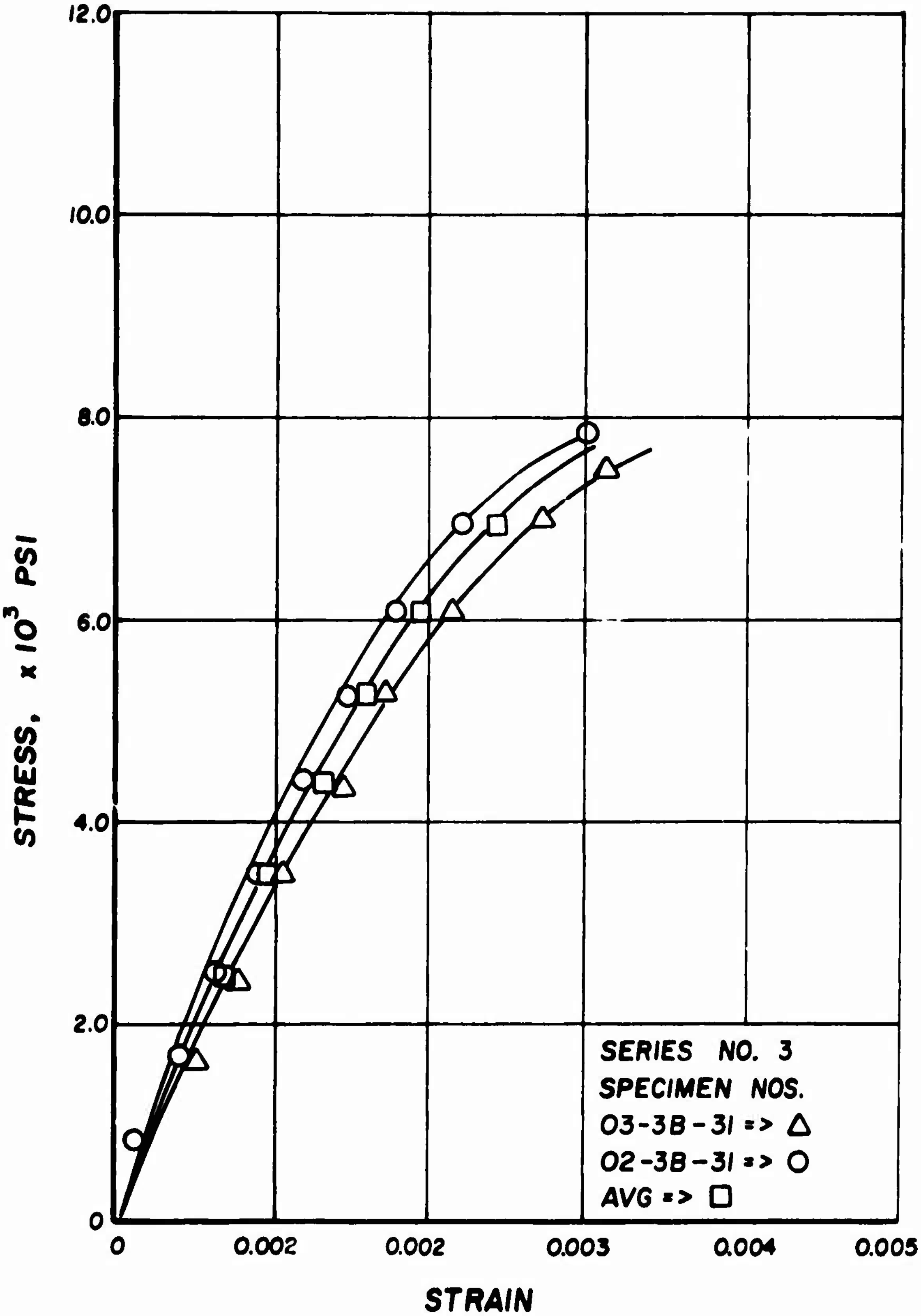


12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 2
SPECIMEN NOS.
26-2C-31 = △
27-2C-31 = ○

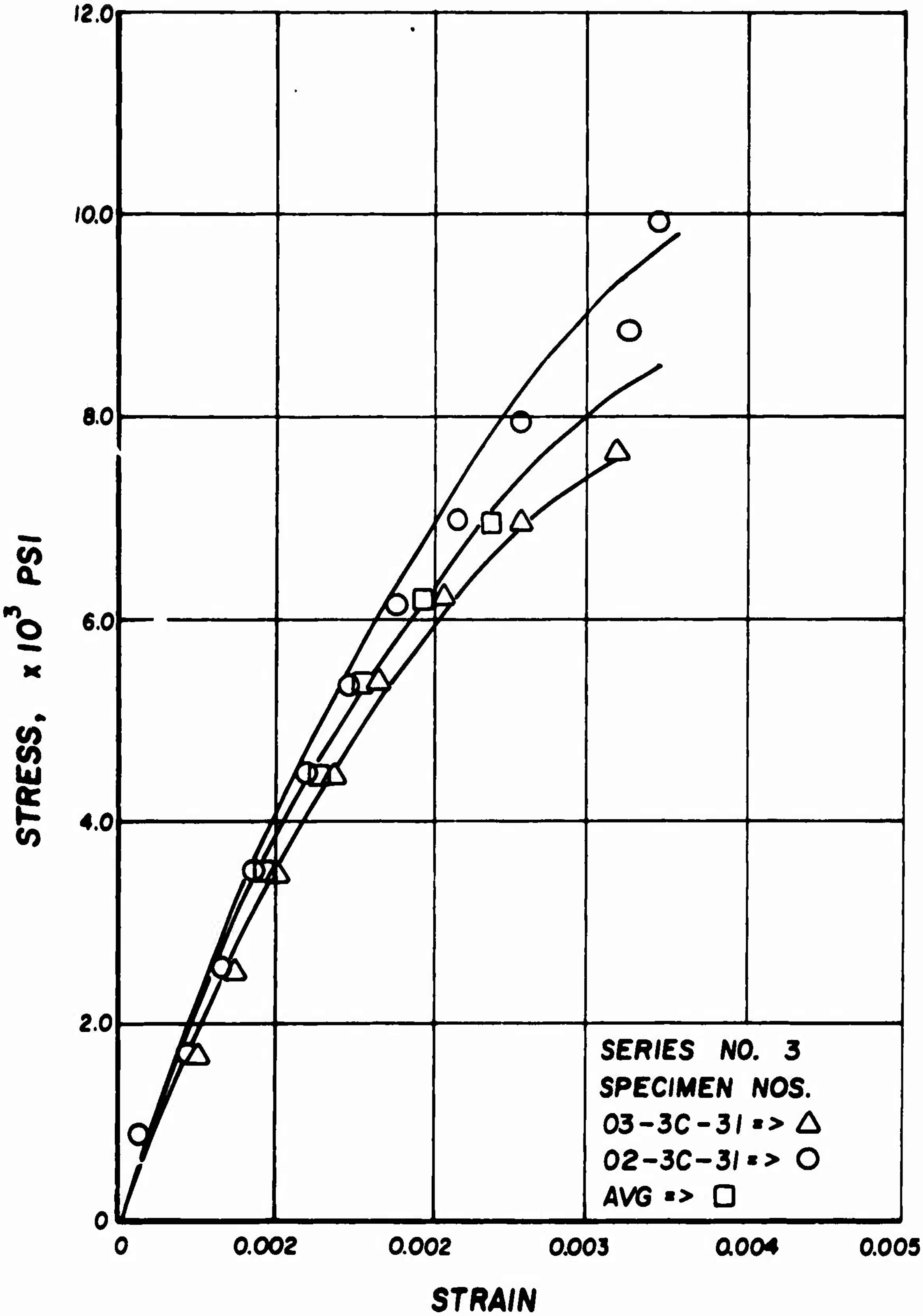

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 2
SPECIMEN NOS.
27-2D-31 => △
26-2D-31 => ○

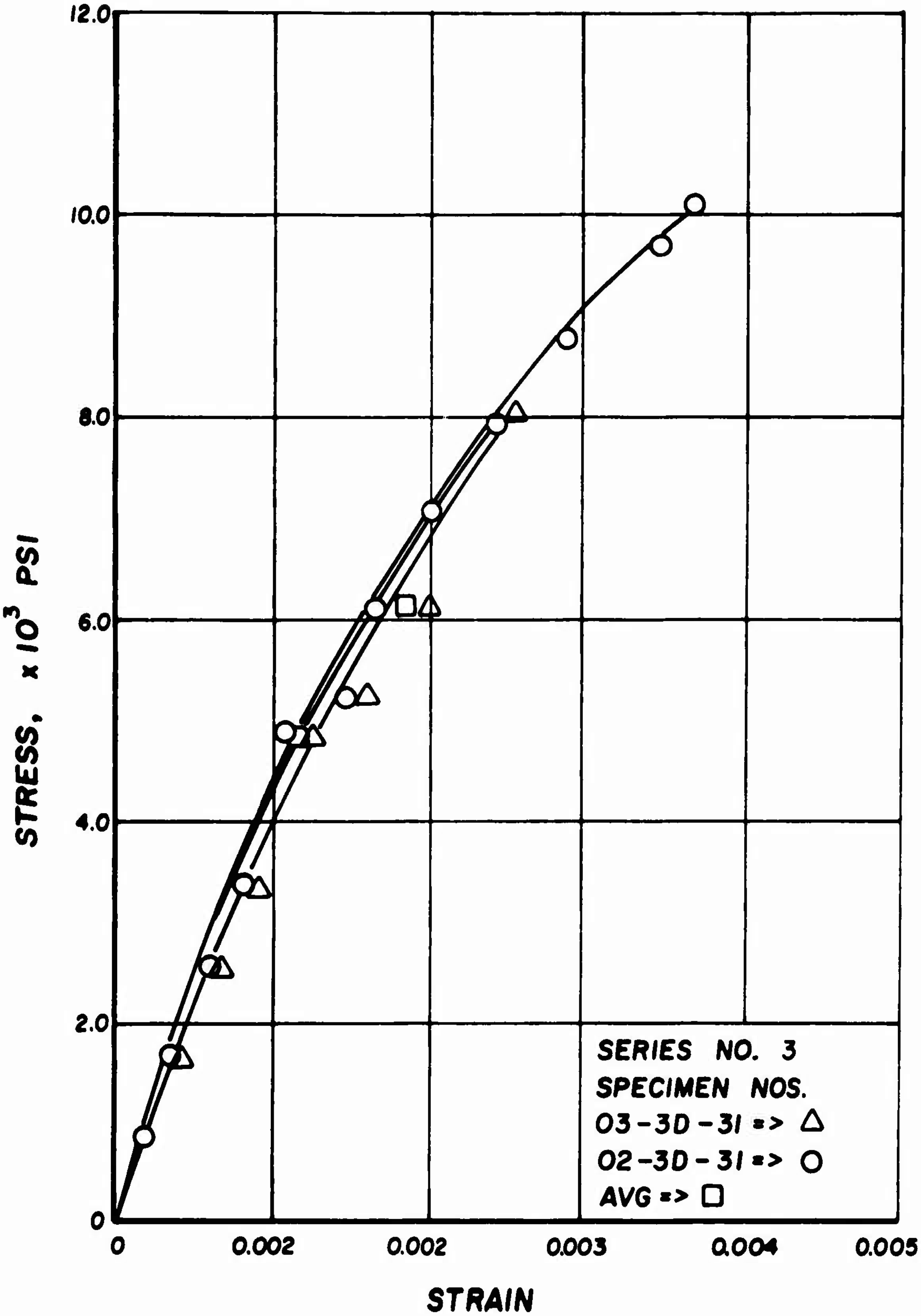

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10³ PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 2
SPECIMEN NOS.
27-2E-31 => △
26-2E-31 => ○

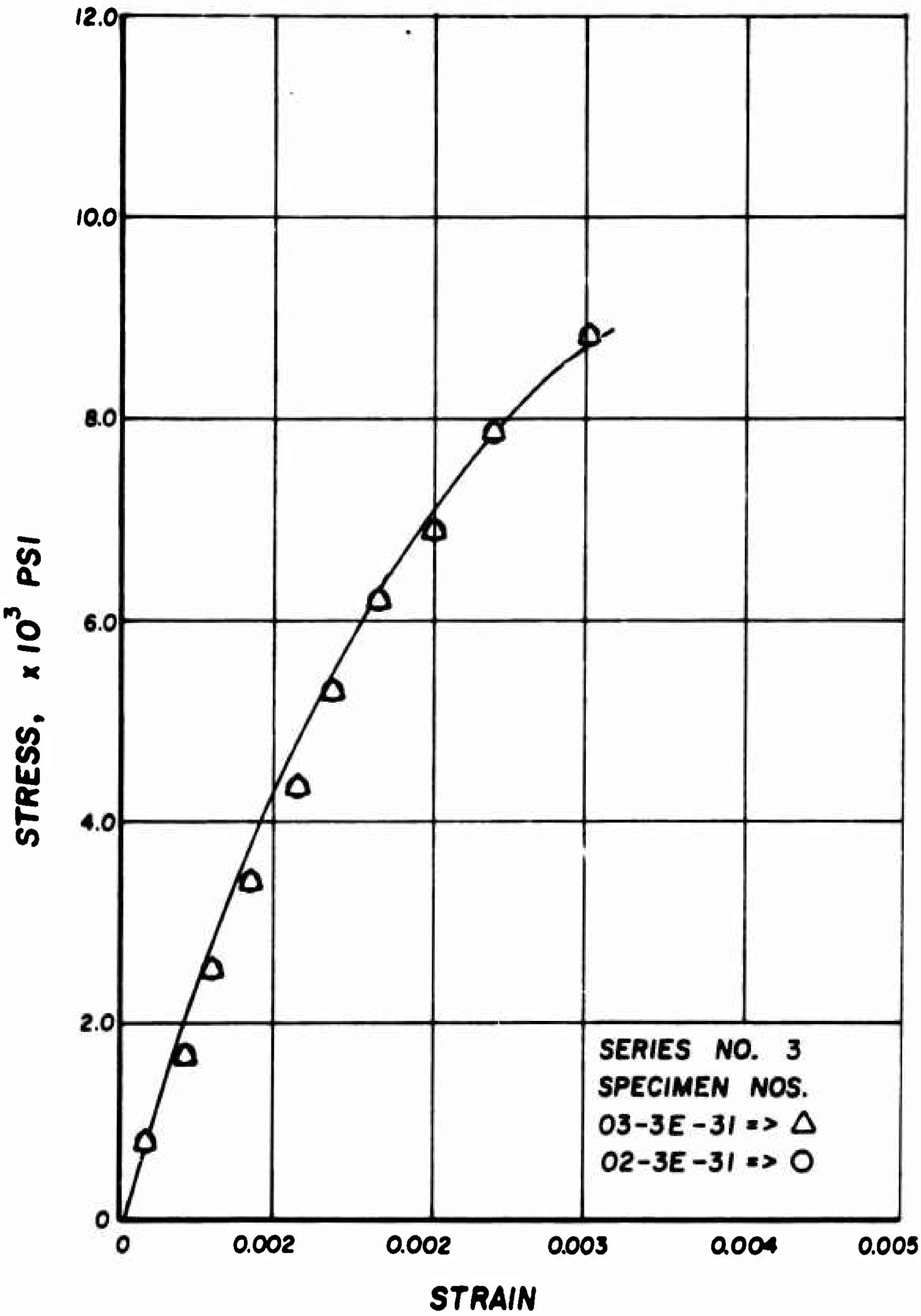

12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 3
SPECIMEN NOS.
03-3A-31 => △
02-3A-31 => ○
AVG => □



12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10³ PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 3
SPECIMEN NOS.
03-3B-31 => △
02-3B-31 => ○
AVG => □


12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 3
SPECIMEN NOS.
03-3C-31 => △
02-3C-31 => ○
AVG => □


12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10^3 PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 3
SPECIMEN NOS.
03-3D-31 => △
02-3D-31 => ○
AVG => □


12.0
10.0
8.0
6.0
4.0
2.0
0
STRESS, x 10³ PSI
0
0.002
0.002
0.003
0.004
0.005
STRAIN
SERIES NO. 3
SPECIMEN NOS.
03-3E-31 => △
02-3E-31 => O